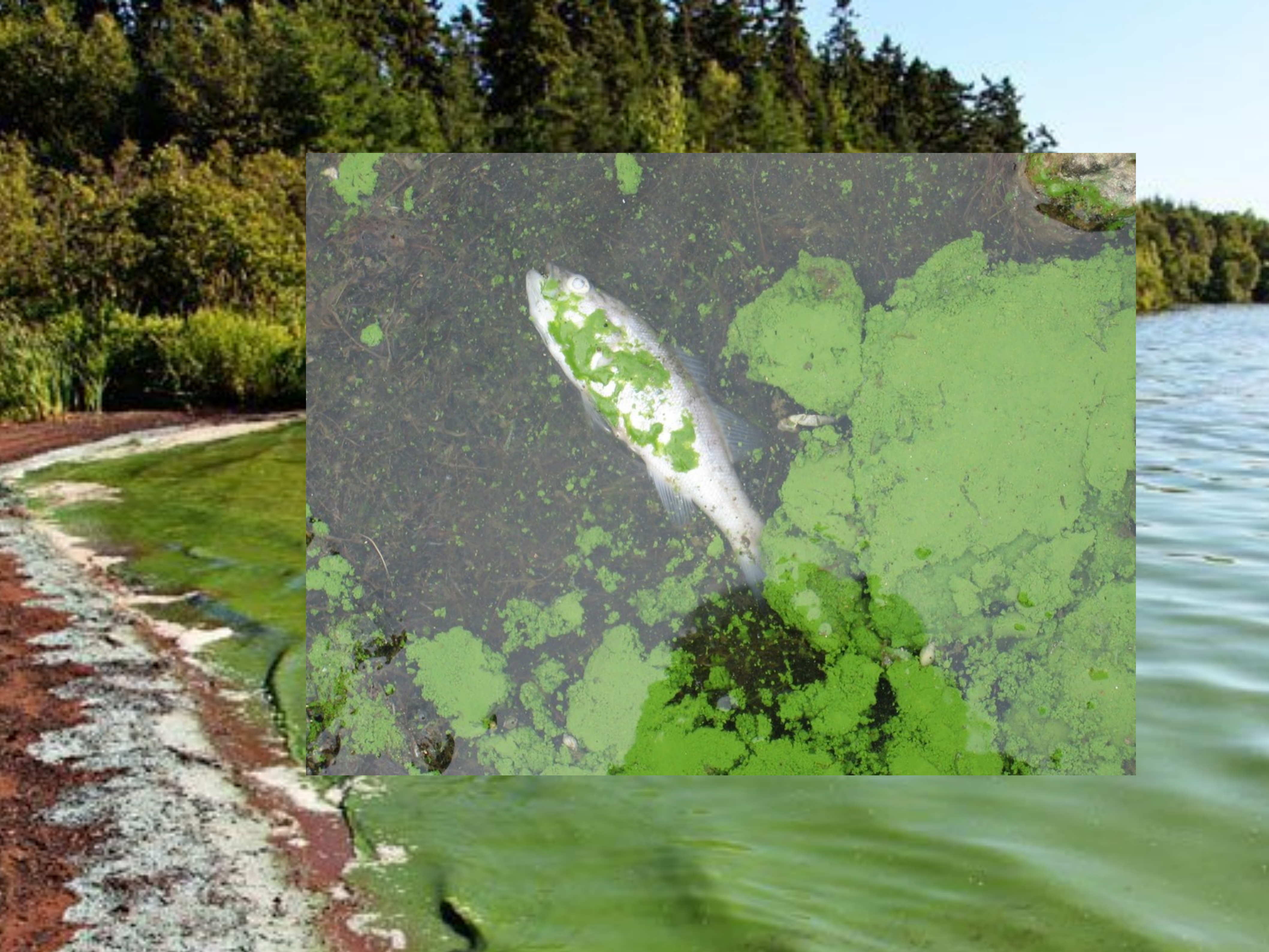
On the stability of large ecological communities

[when an ecosystem breaks]

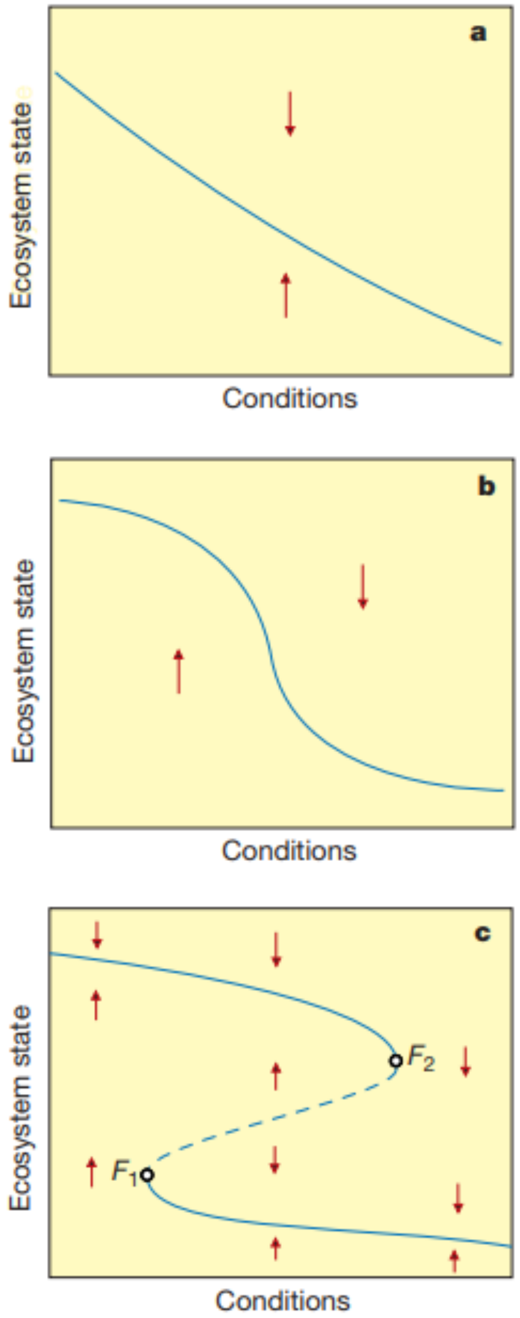
Jacopo Grilli

Santa Fe Institute

WG Cognitive Regime Shift I, SFI, July 23rd 2018

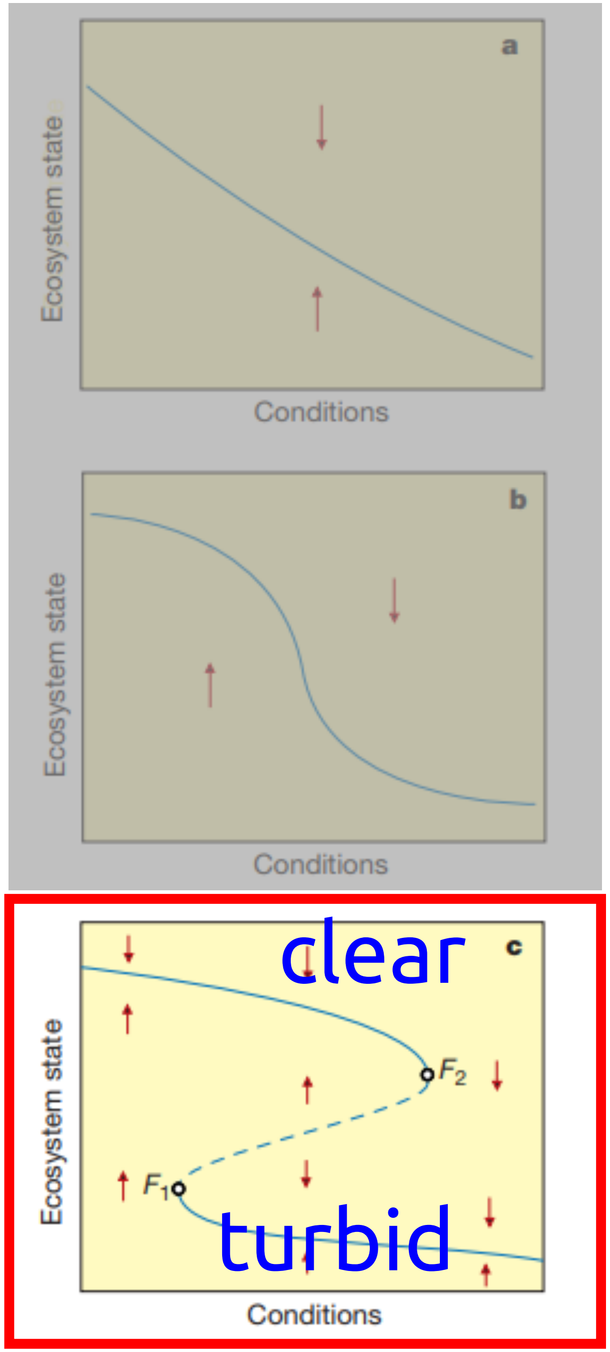


Regime shifts



[Scheffer et al., Nature 2001]

Regime shifts



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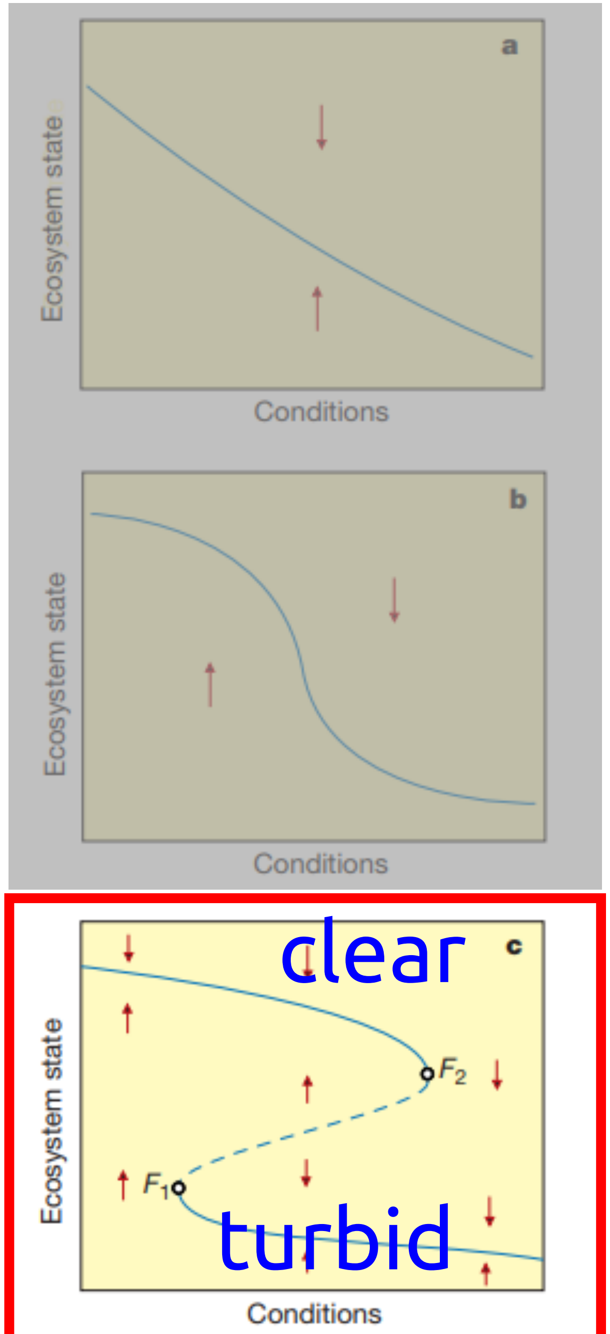
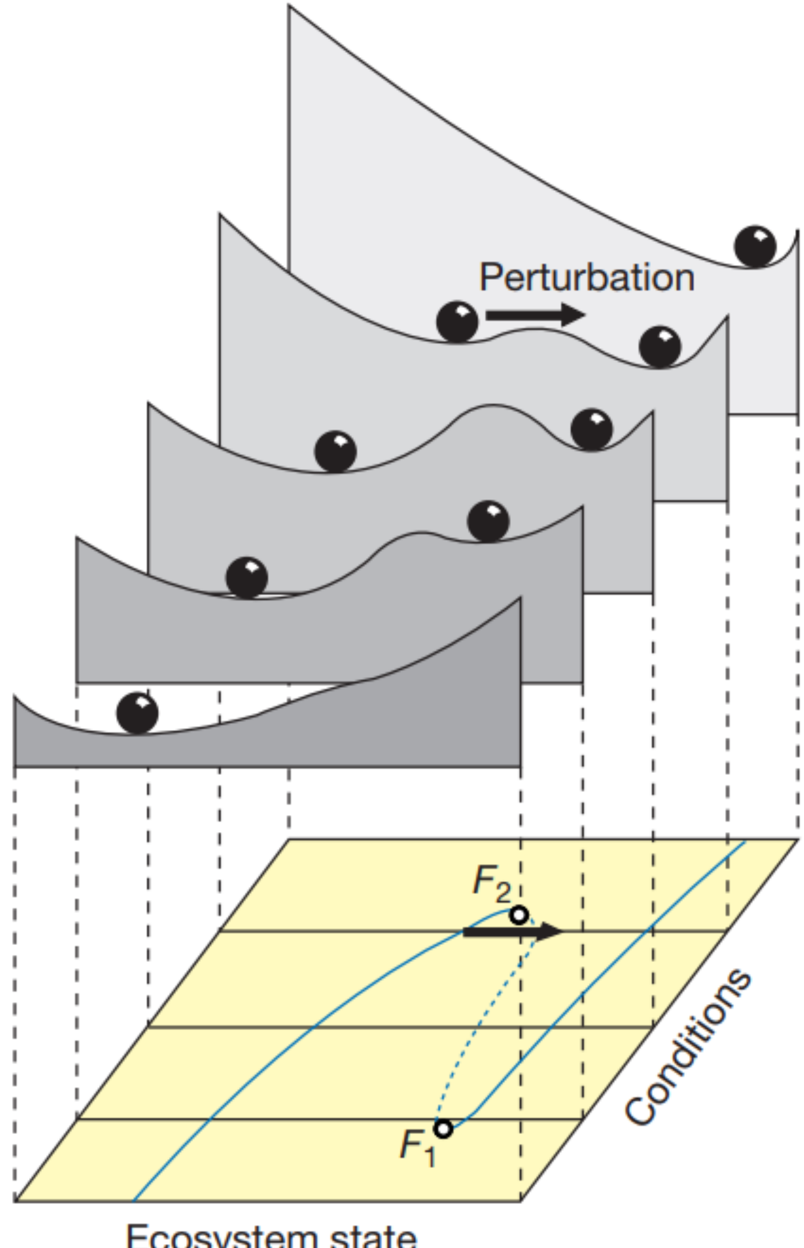
Regime shifts





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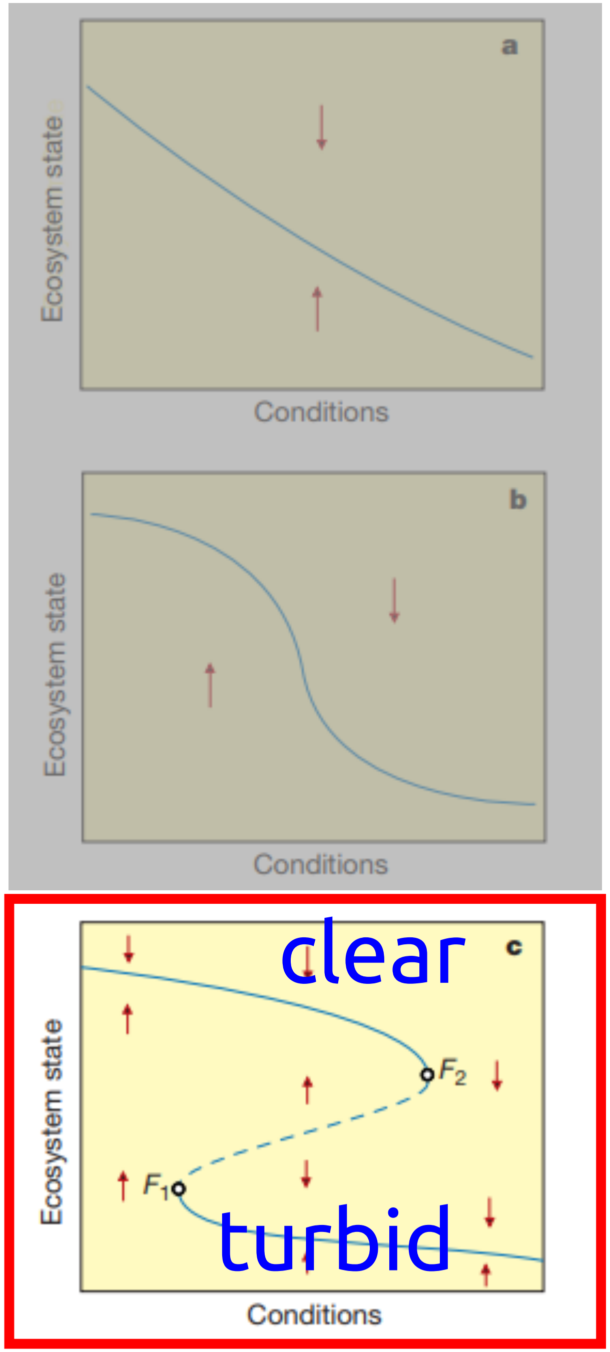
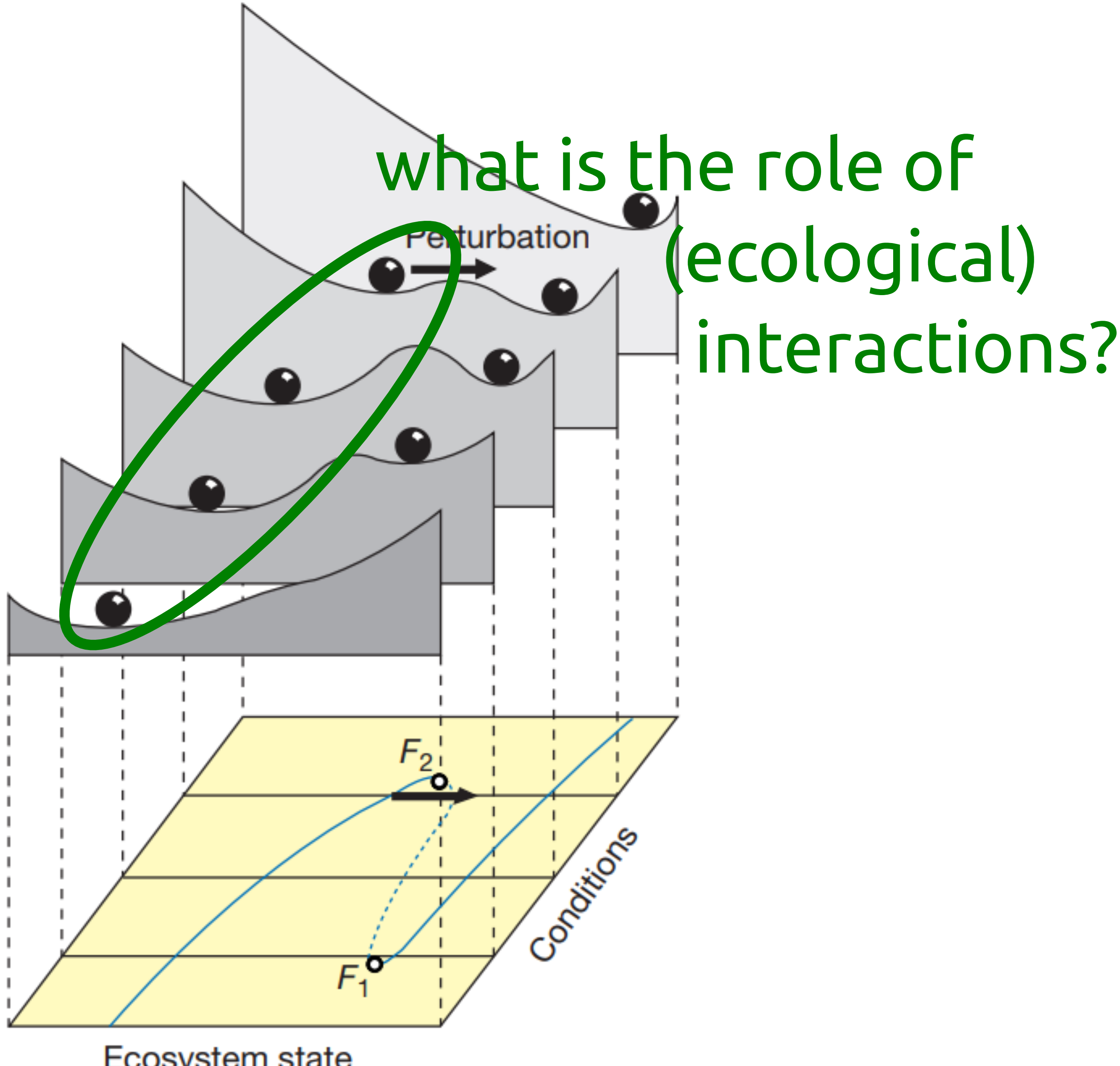
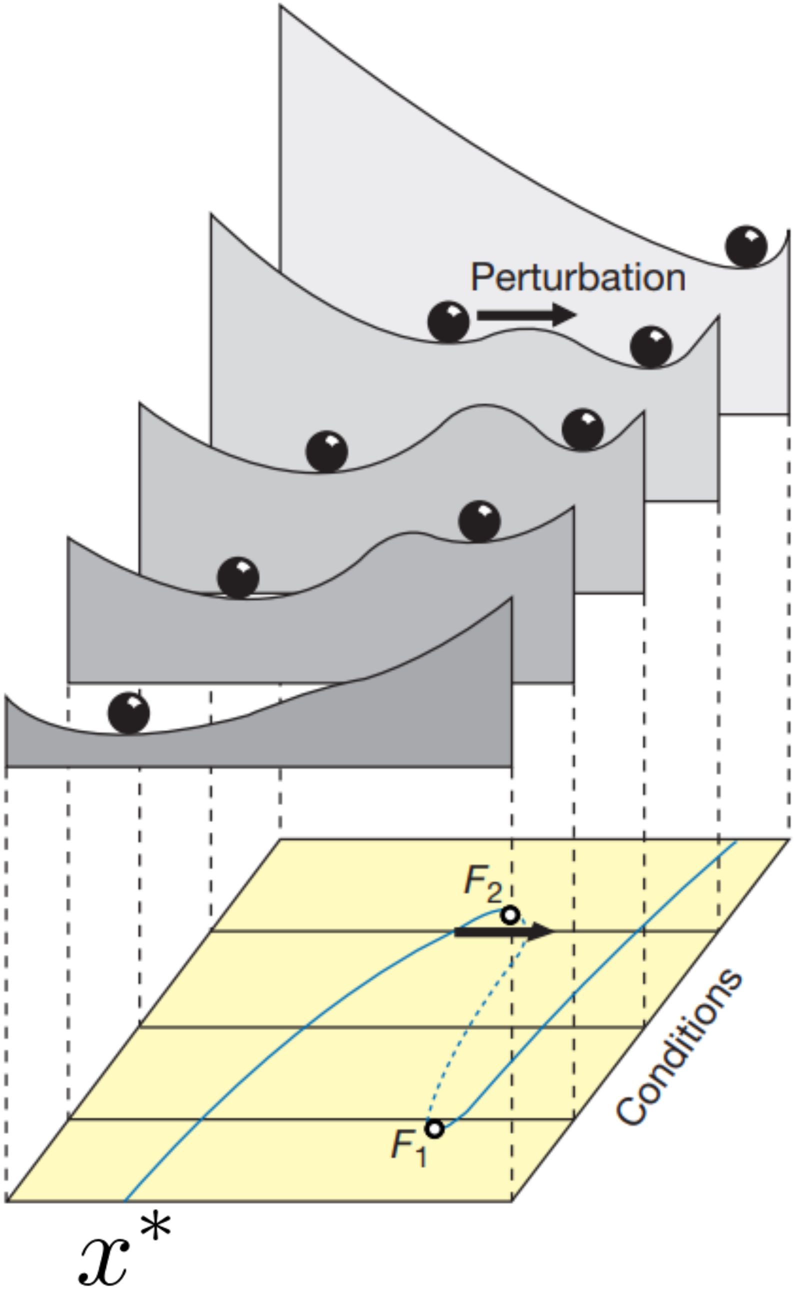




[Scheffer et al., Nature 2001]

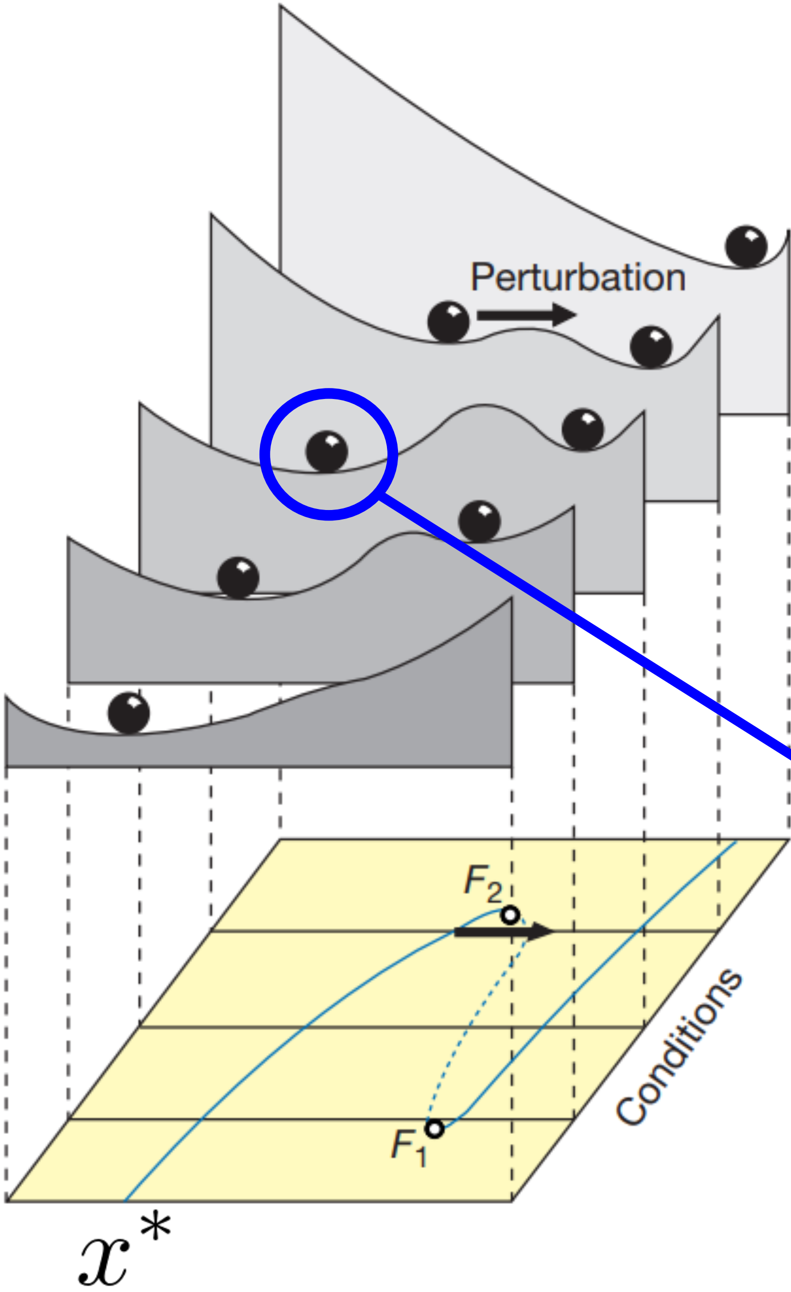
Resilience and stability



$$\frac{dx_i}{dt} = f_i(\underline{x}) + \text{perturbations}$$

$$0 = f_i(\underline{x}^*)$$

Resilience and stability



$$\frac{dx_i}{dt} = f_i(\underline{x}) + \text{perturbations}$$

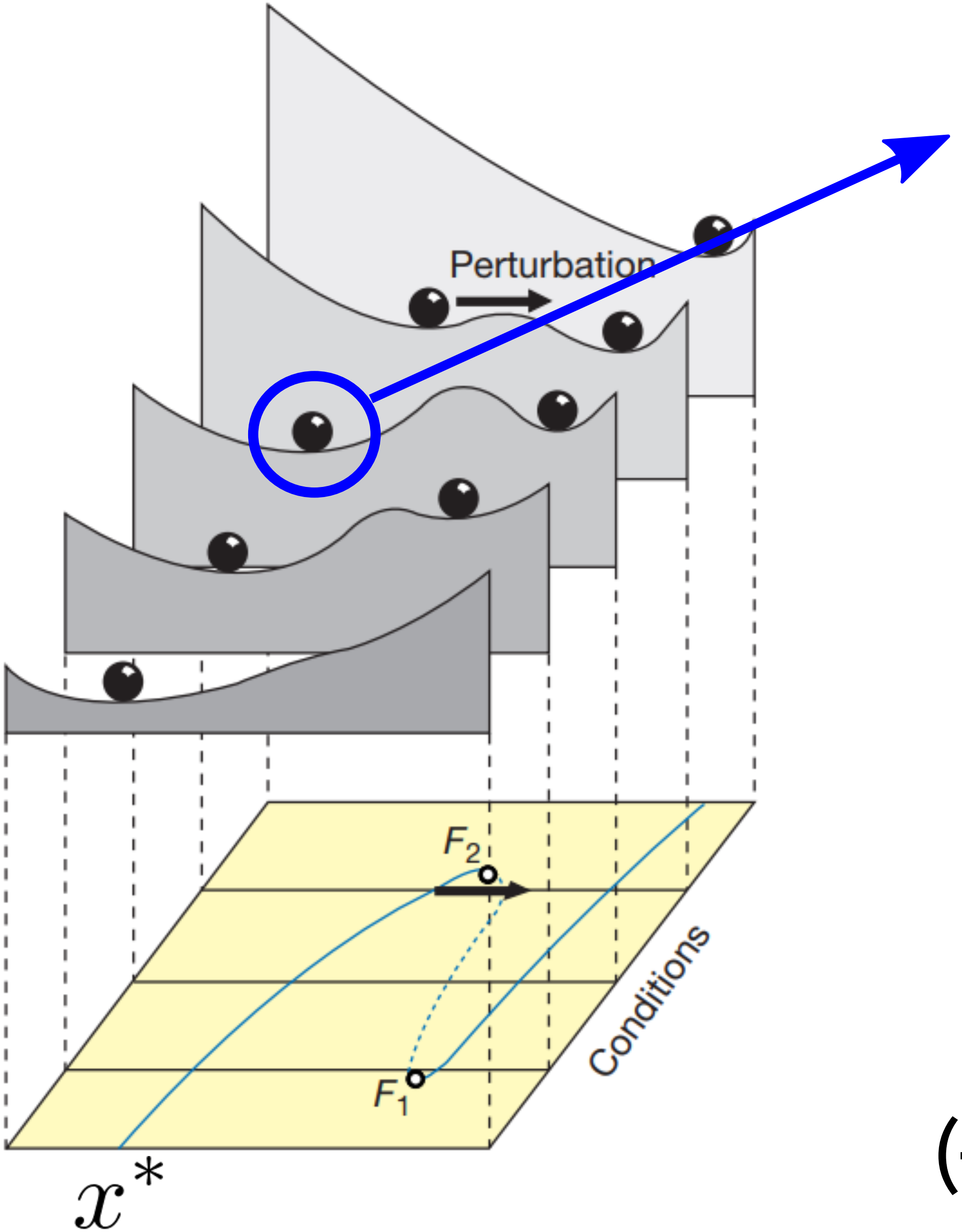
$$0 = f_i(\underline{x}^*)$$

linearize (~small perturbations)

$$\delta x_i = x_i - x_i^* \qquad A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\underline{x}^*}$$

$$\frac{d\delta x_i}{dt} = \sum_j A_{ij}\delta x_j$$

Resilience and stability



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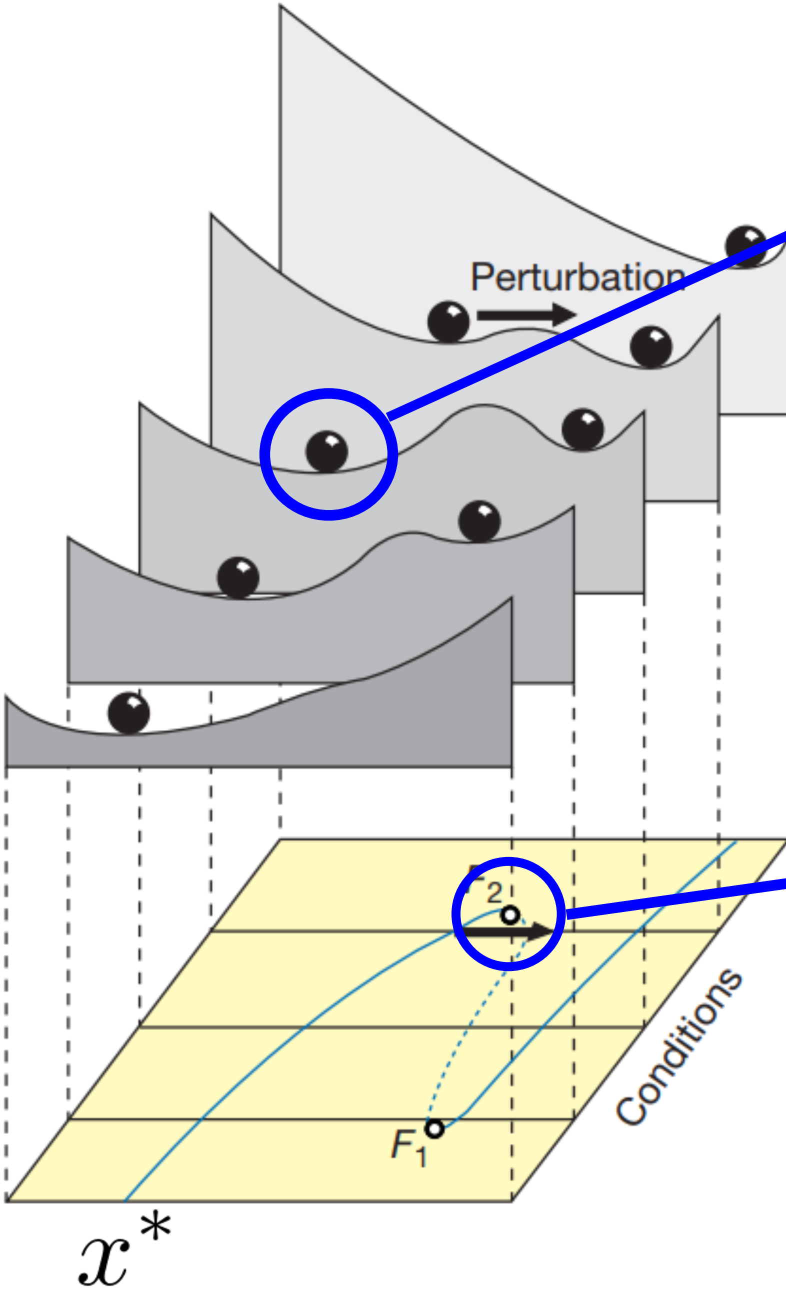
(-λ* determines the speed to return to equilibrium)

Resilience and stability



$$\frac{d\delta x_i}{dt} = \sum_j A_{ij}\delta x_j$$

λ* (largest eigenvalue of A)

λ*<0 stable

λ*>0 unstable

λ*=0 critical

(-λ* determines the speed to return to equilibrium)

Random matrix approach

$$\frac{dx_i}{dt} = f_i(\underline{x})$$

$$A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\underline{x}^*}$$

Random matrix approach

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$$A_{ij} = \frac{\partial f_i}{\partial x_j}\bigg|_{\underline{x}^*}$$

Random matrix approach

$$\cancel{\frac{dx_i}{dt} = f_i(\underline{x})}$$

$$\cancel{A_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\underline{x}^*}}$$

model A
- A is large
- A is random (some stochastic rule to fill its entries)

Random matrix approach

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$$A_{ij} = \frac{\partial f_i}{\partial x_j}\Big|_{\underline{x}^*}$$

model A

- A is large
- A is random (some stochastic rule to fill its entries)

the question:

what is the largest eigenvalue of A?

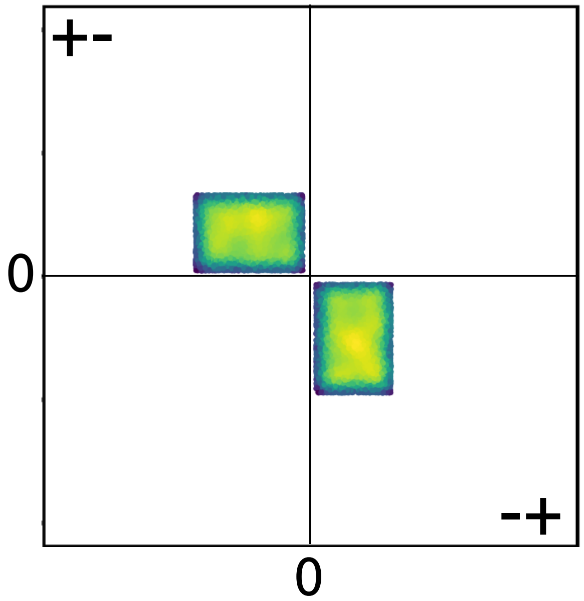
$p(A_{ij},A_{ji})$

Why large and random?

Why large and random?

$p(A_{ij},A_{ji})$

30% pairs are interacting



Why large and random?

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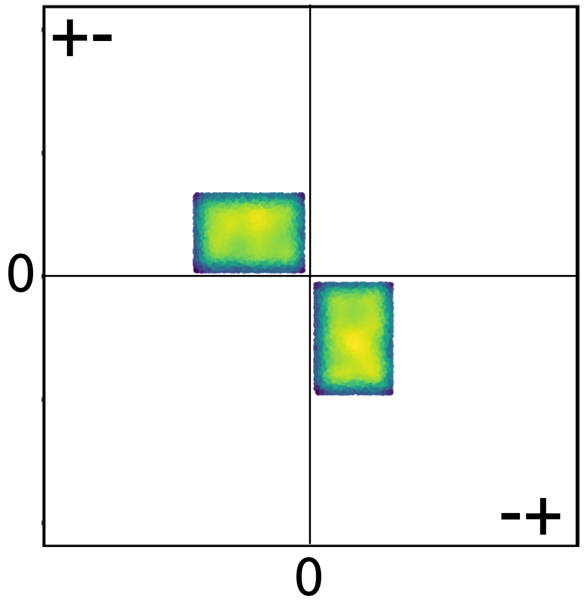

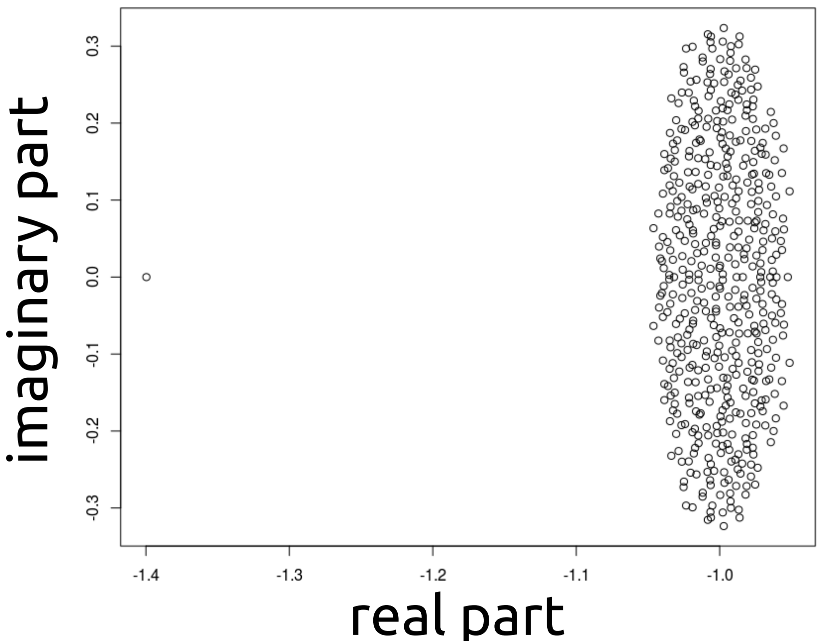

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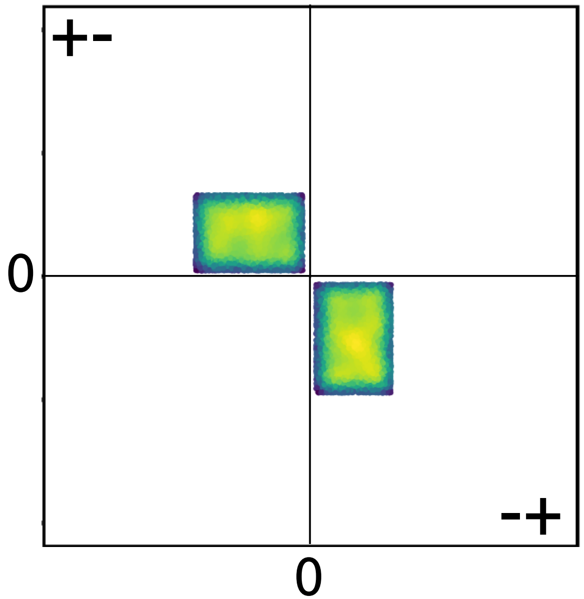
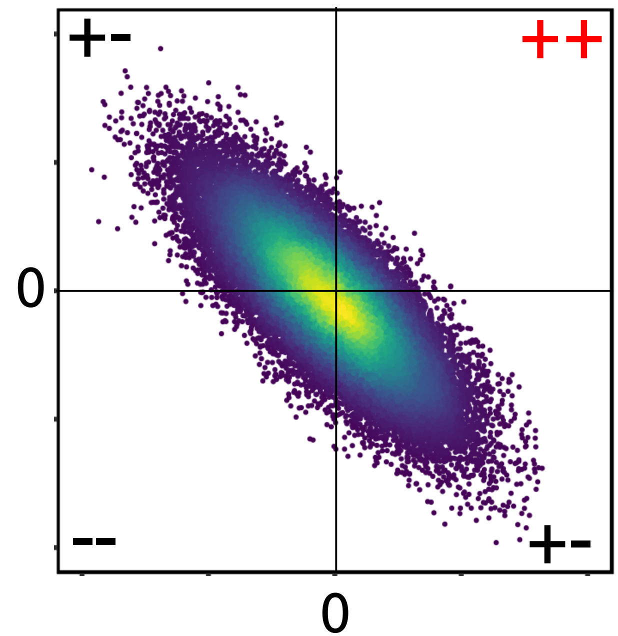
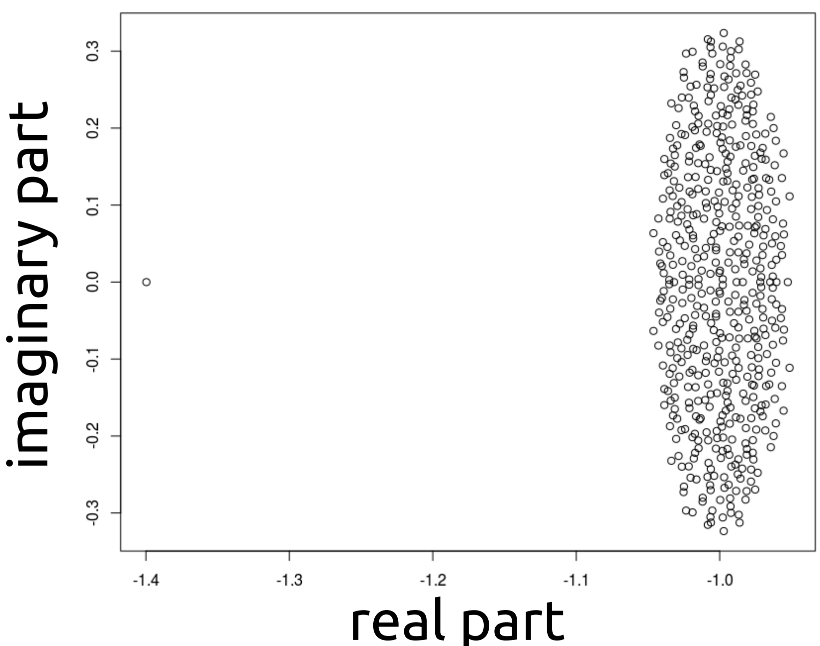


100% pairs are interacting





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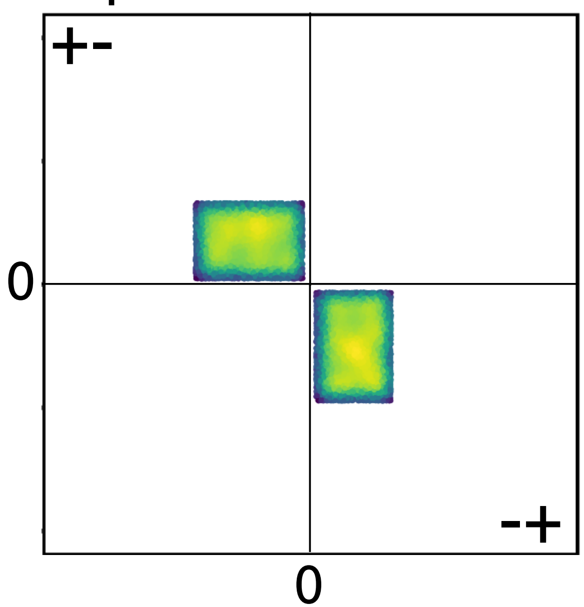

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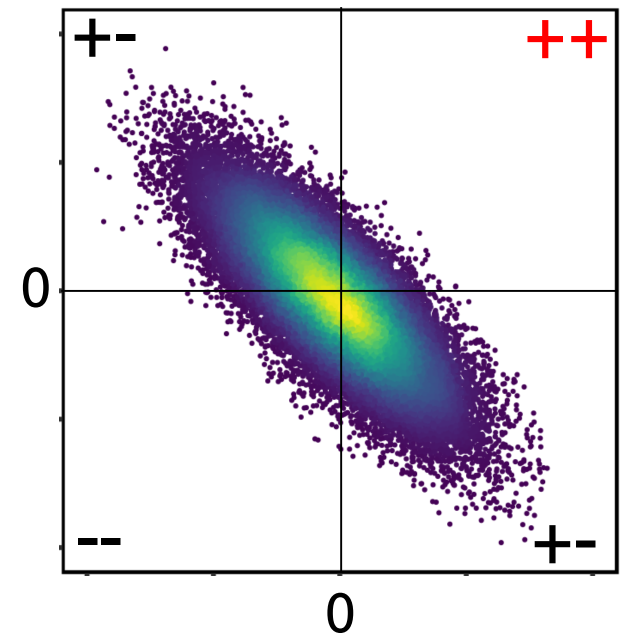
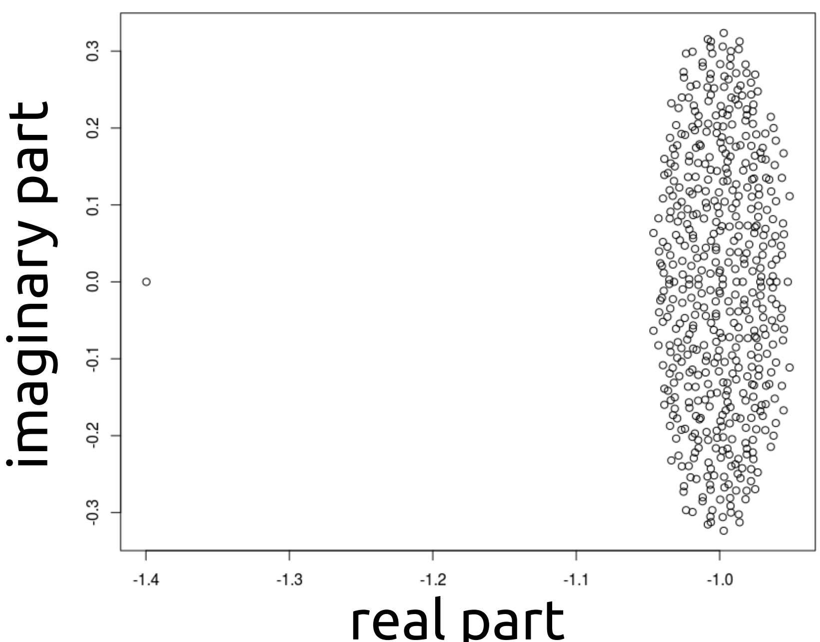
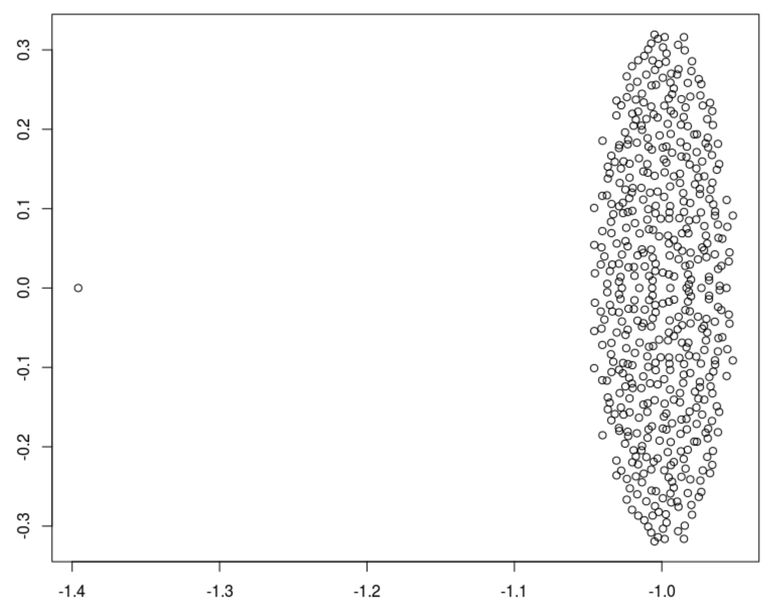
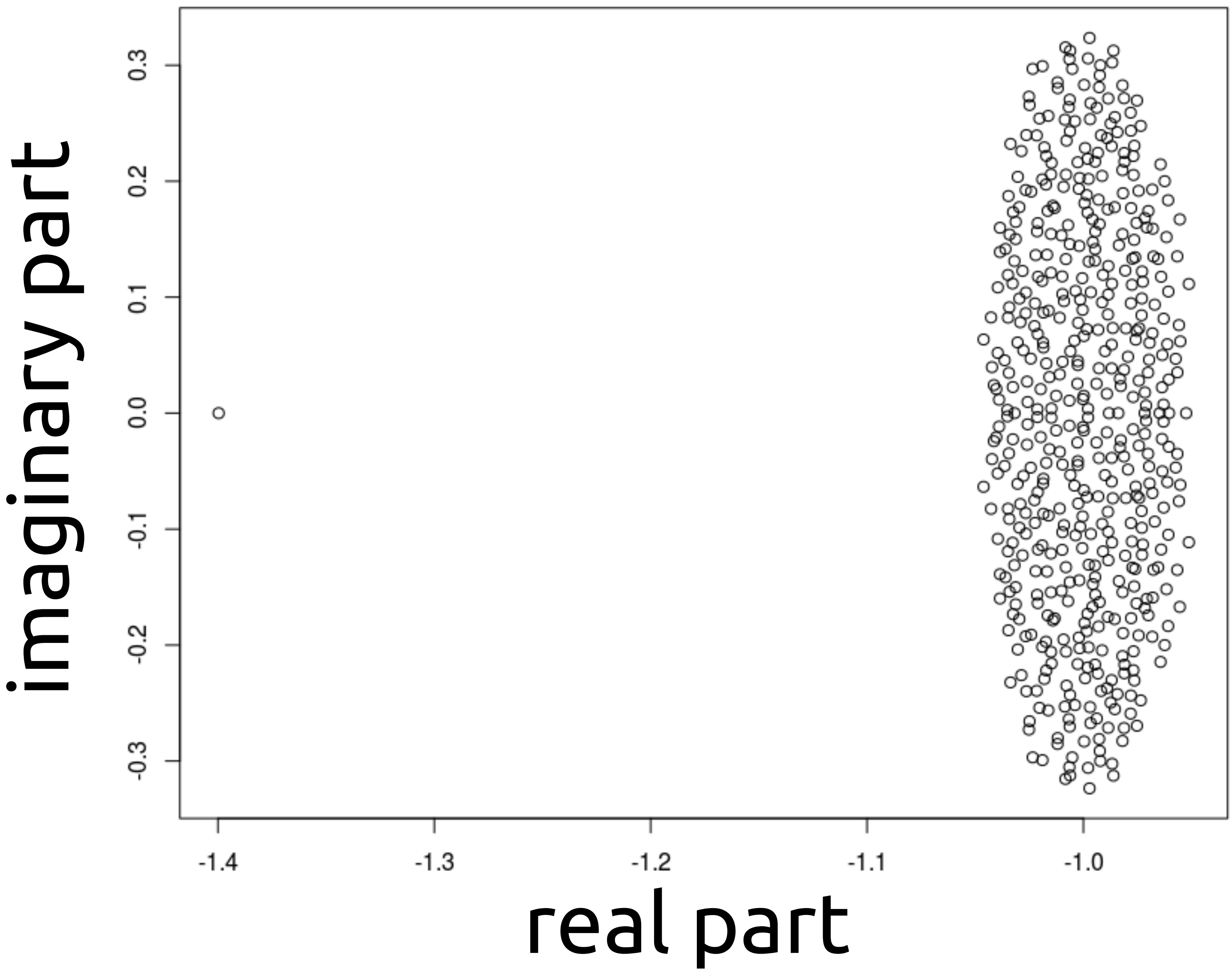






Anatomy of a random matrix



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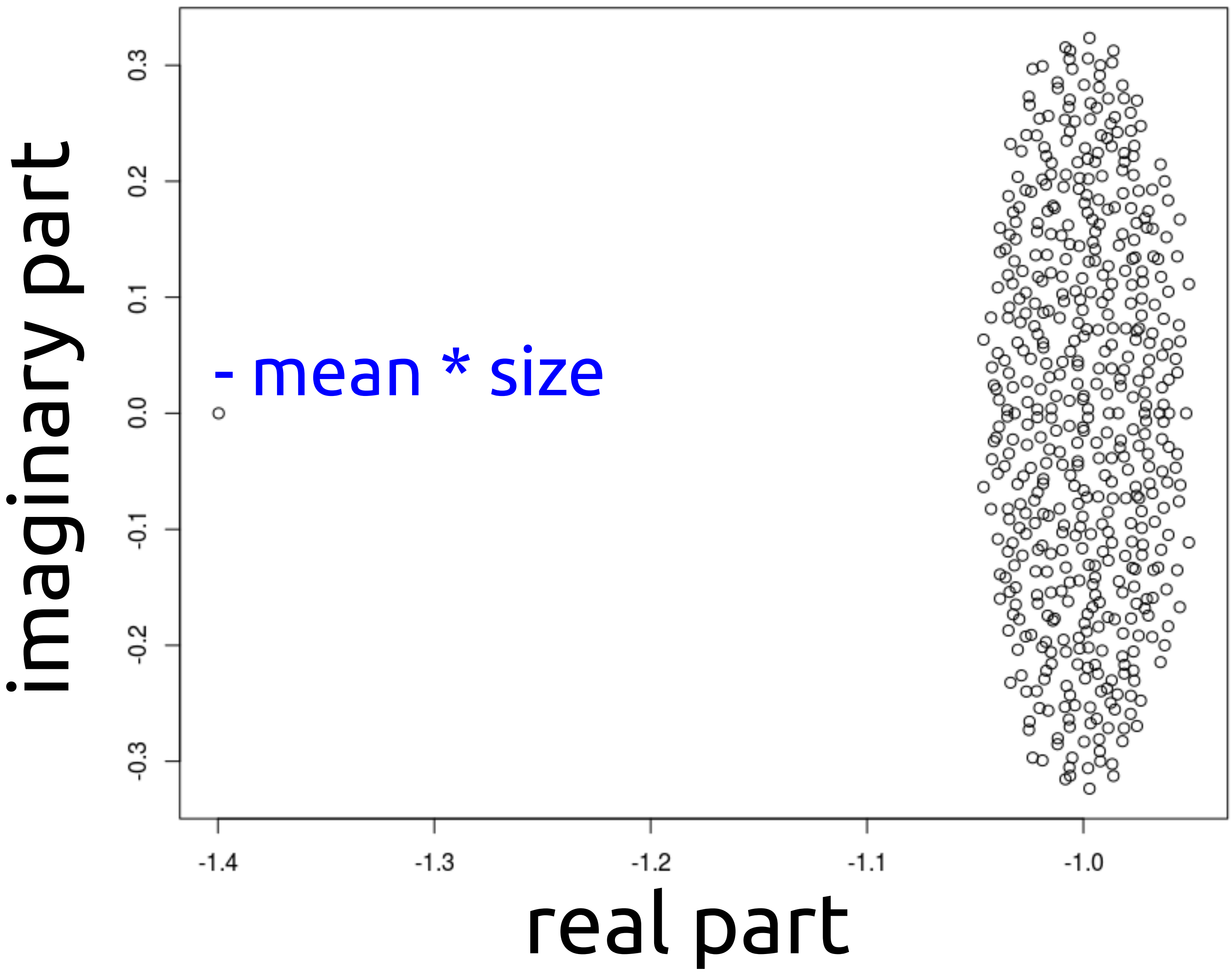

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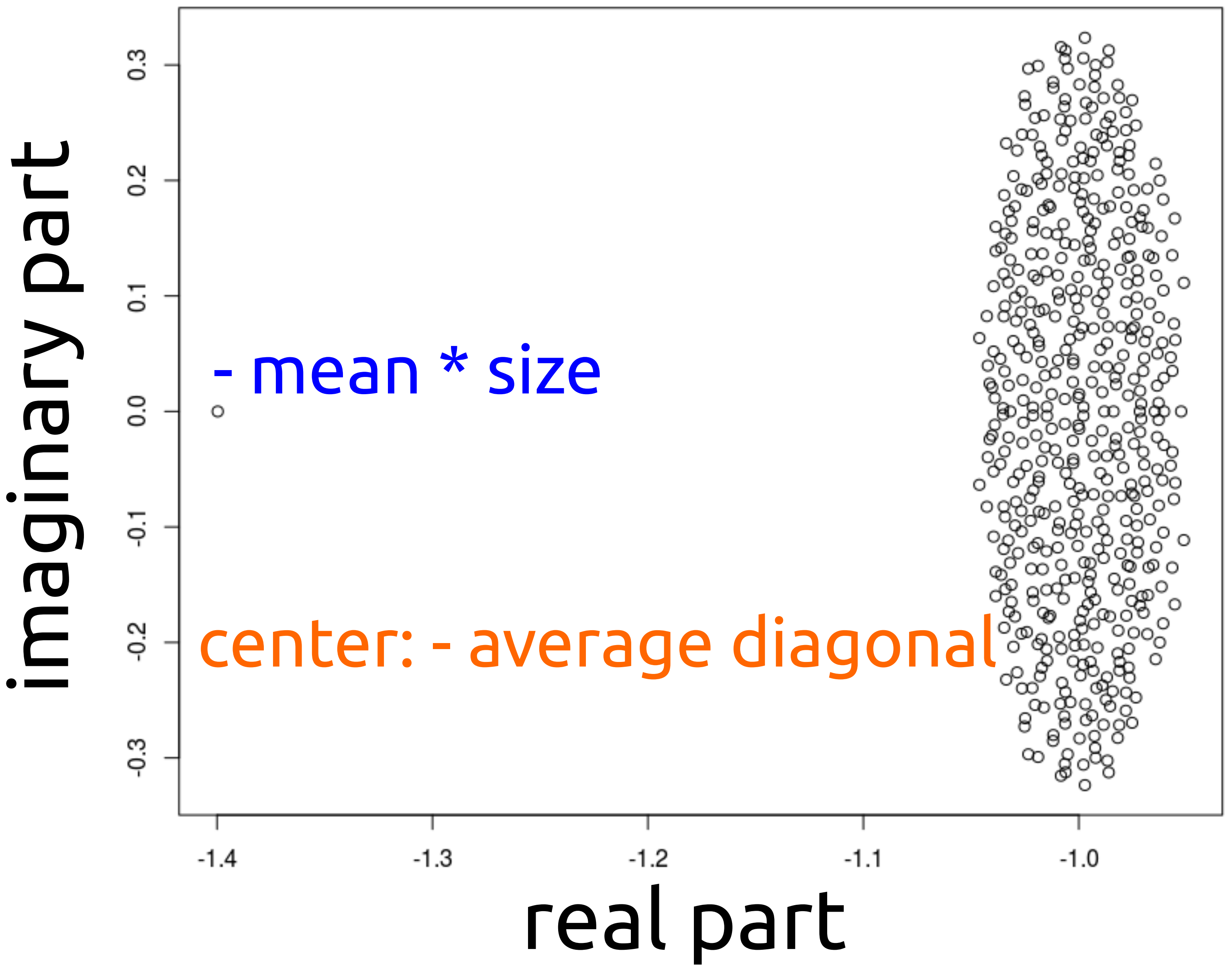

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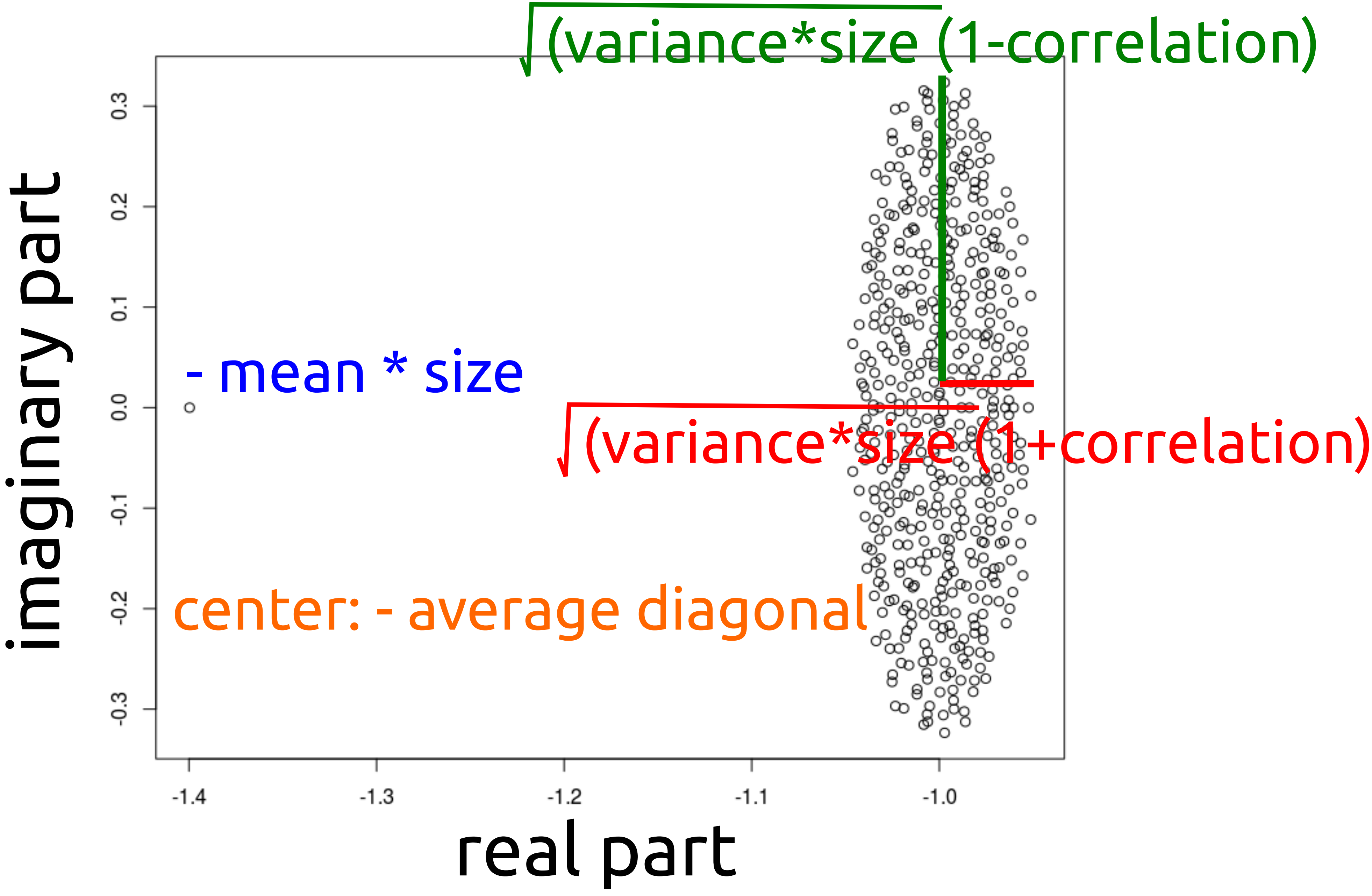

If the interactions are random

- only 4 important parameters (instead of size^2)
- a realization behaves as the average

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four examples:
- directionality
- modules / communities
- effect of the fixed point
- space

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[Allesina, Grilli, Barbaas, Tang, Maritan, Nature Communications, 2015]

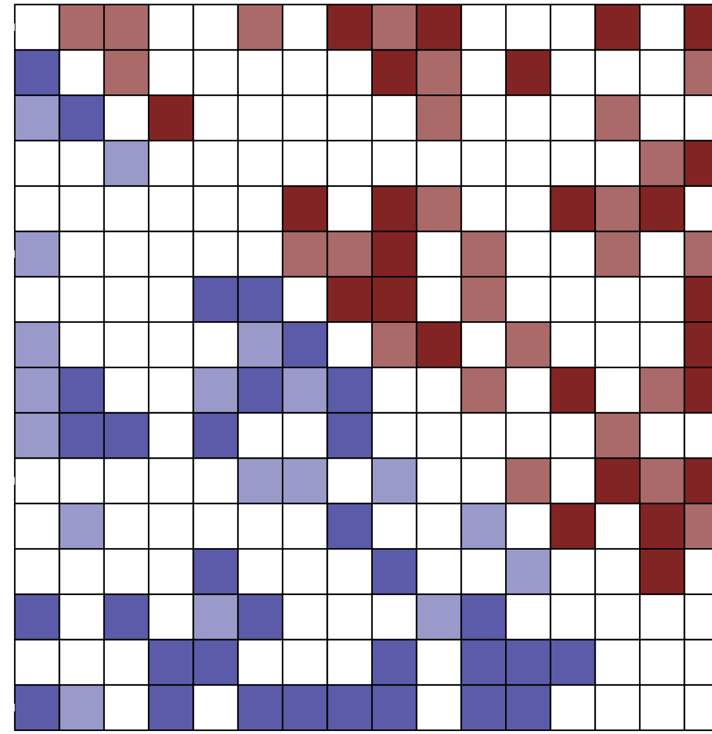
Pieter Bruegel the Elder, 1557

Cascade model

Big Fish Eat Little Fish

Pieter Bruegel the Elder, 1557

Cascade model



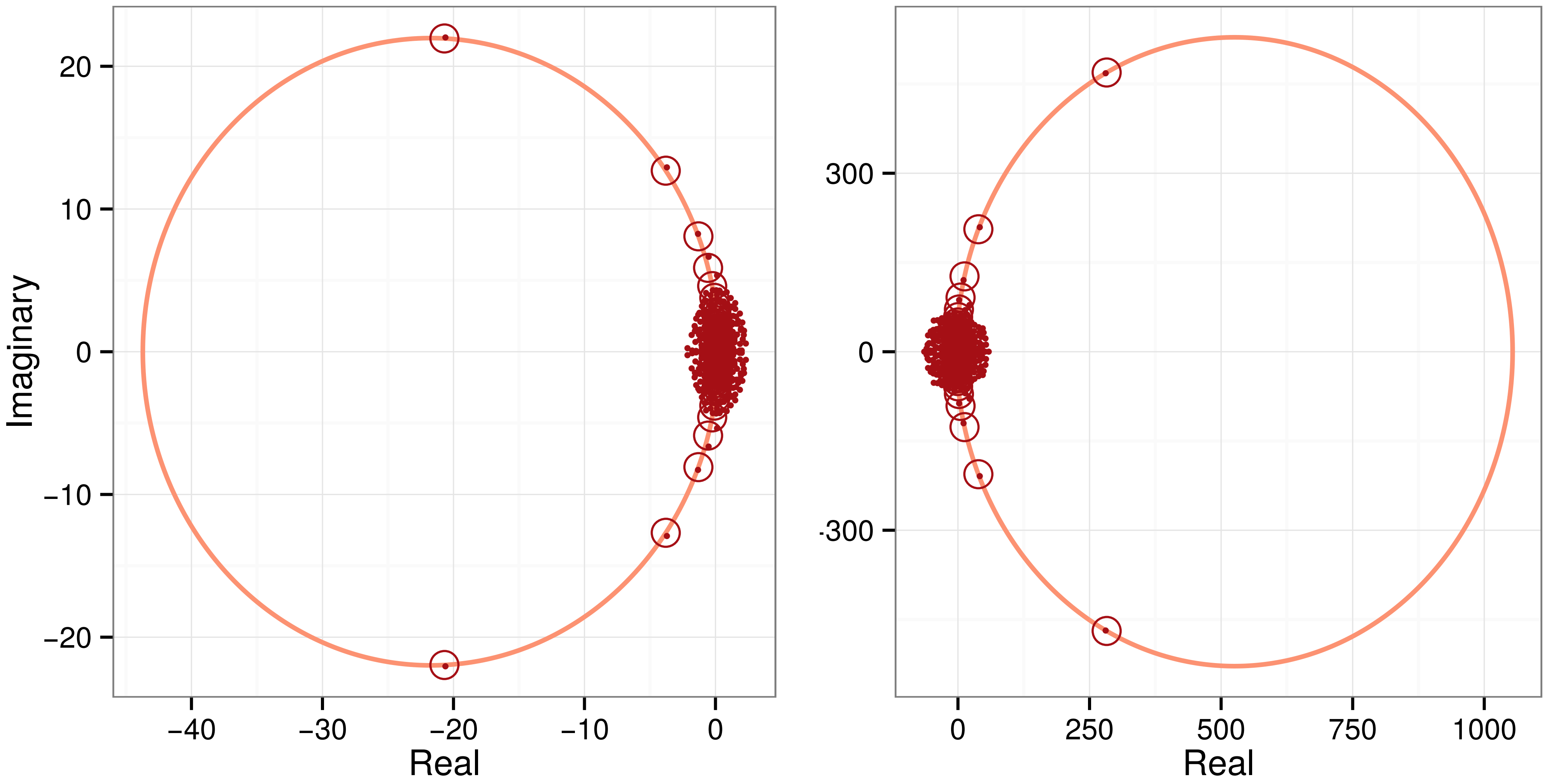
Cohen et al., 1990

order S species

species i has probability C of eating any of the preceding species

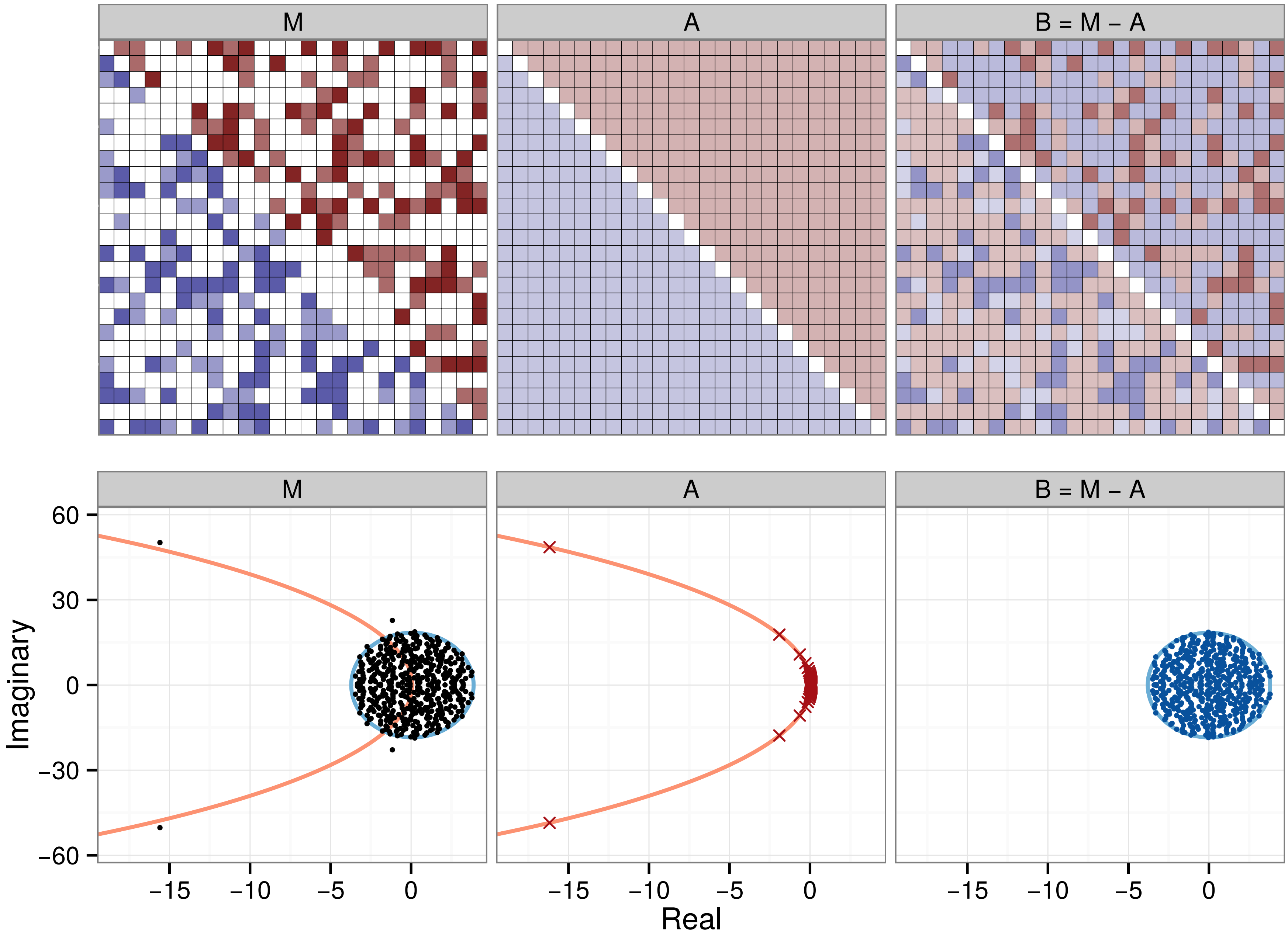
produces acyclic graphs

The eyeball

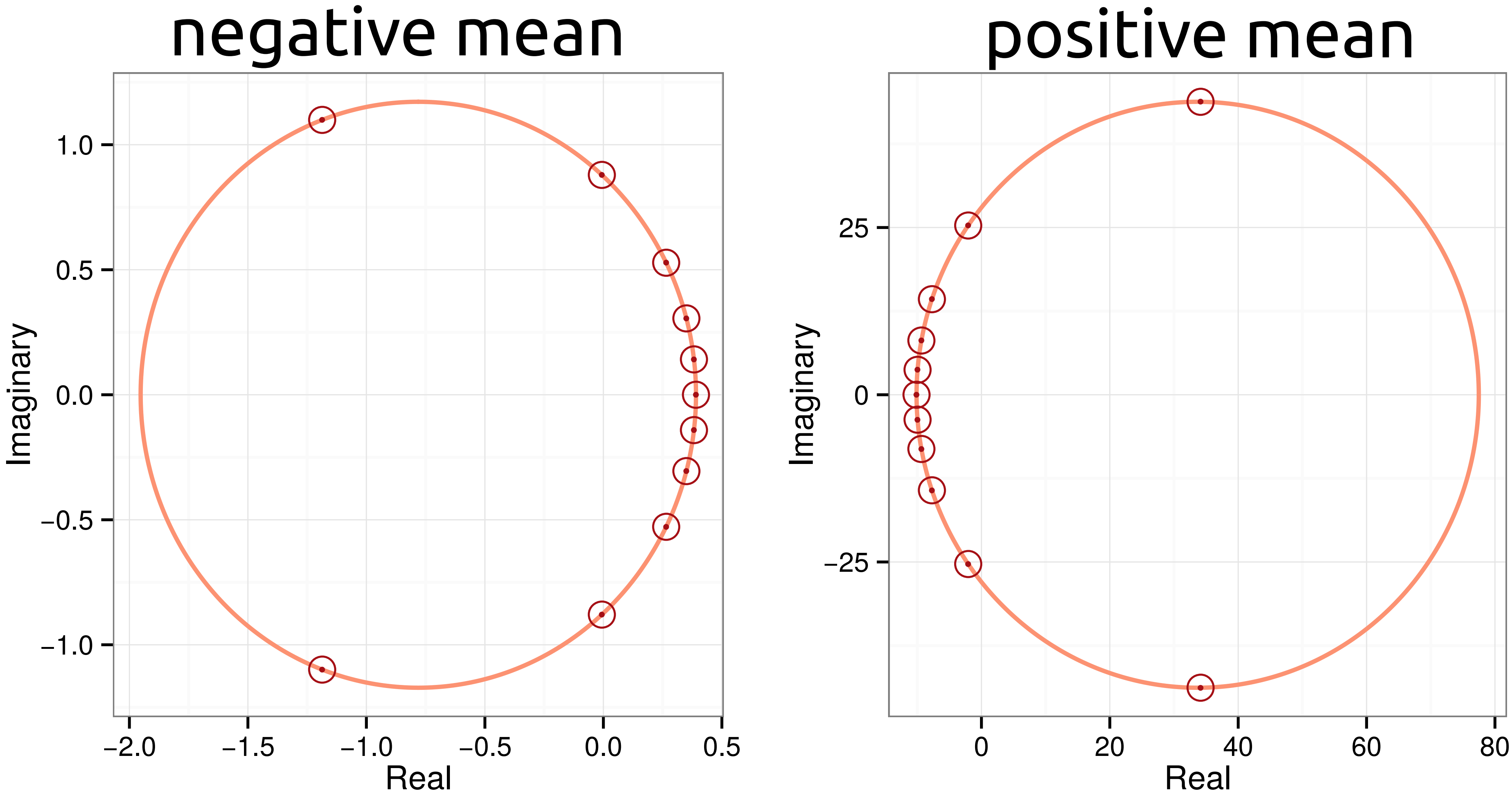


20
10
0
−10
−20
Imaginary
−40
−30
−20
−10
0
Real
300
0
-300
0
250
500
750
1000
Real

Our strategy: eyeball = eye + ball

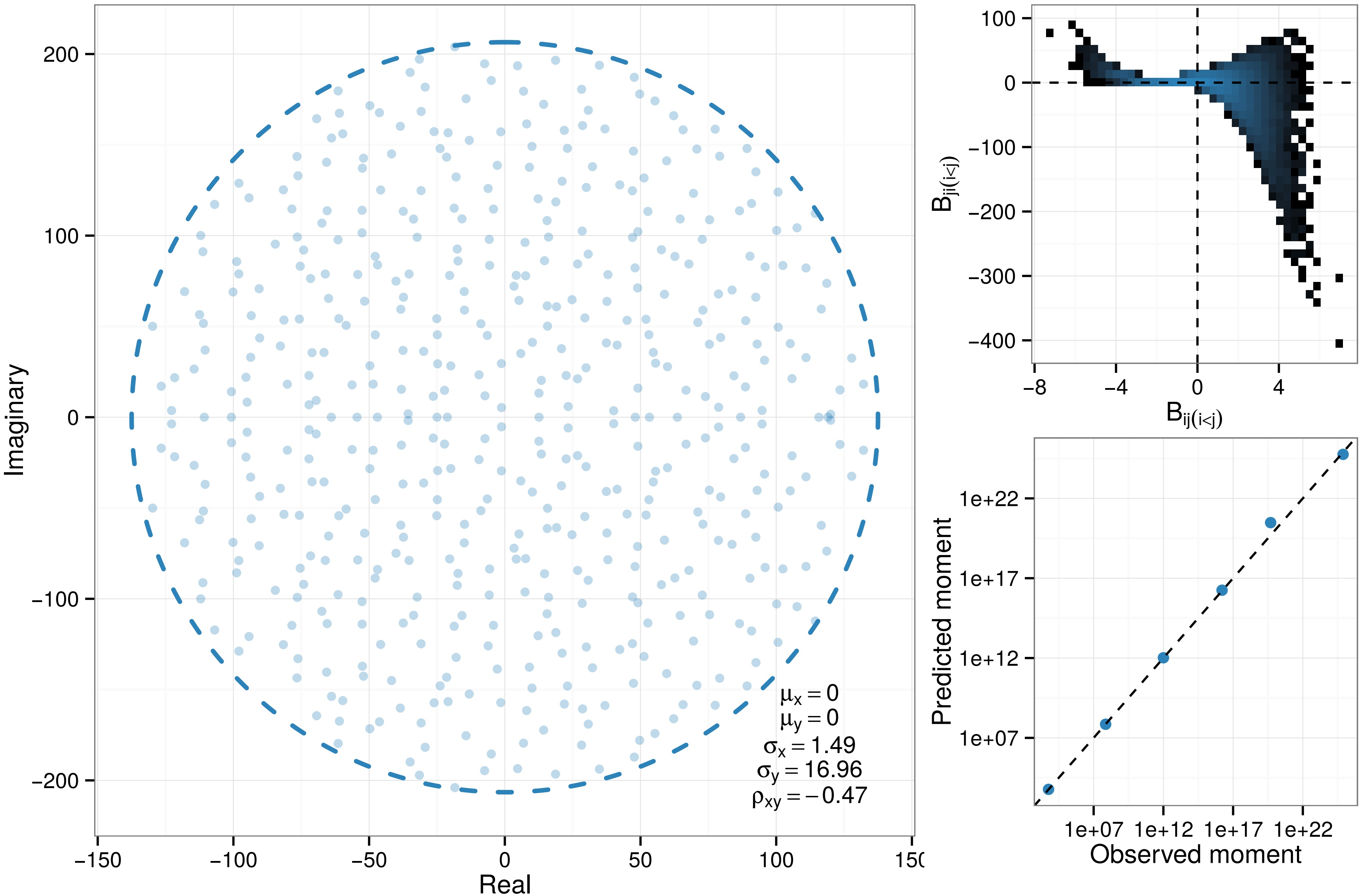


Eigenvalues of A lay on a circumference

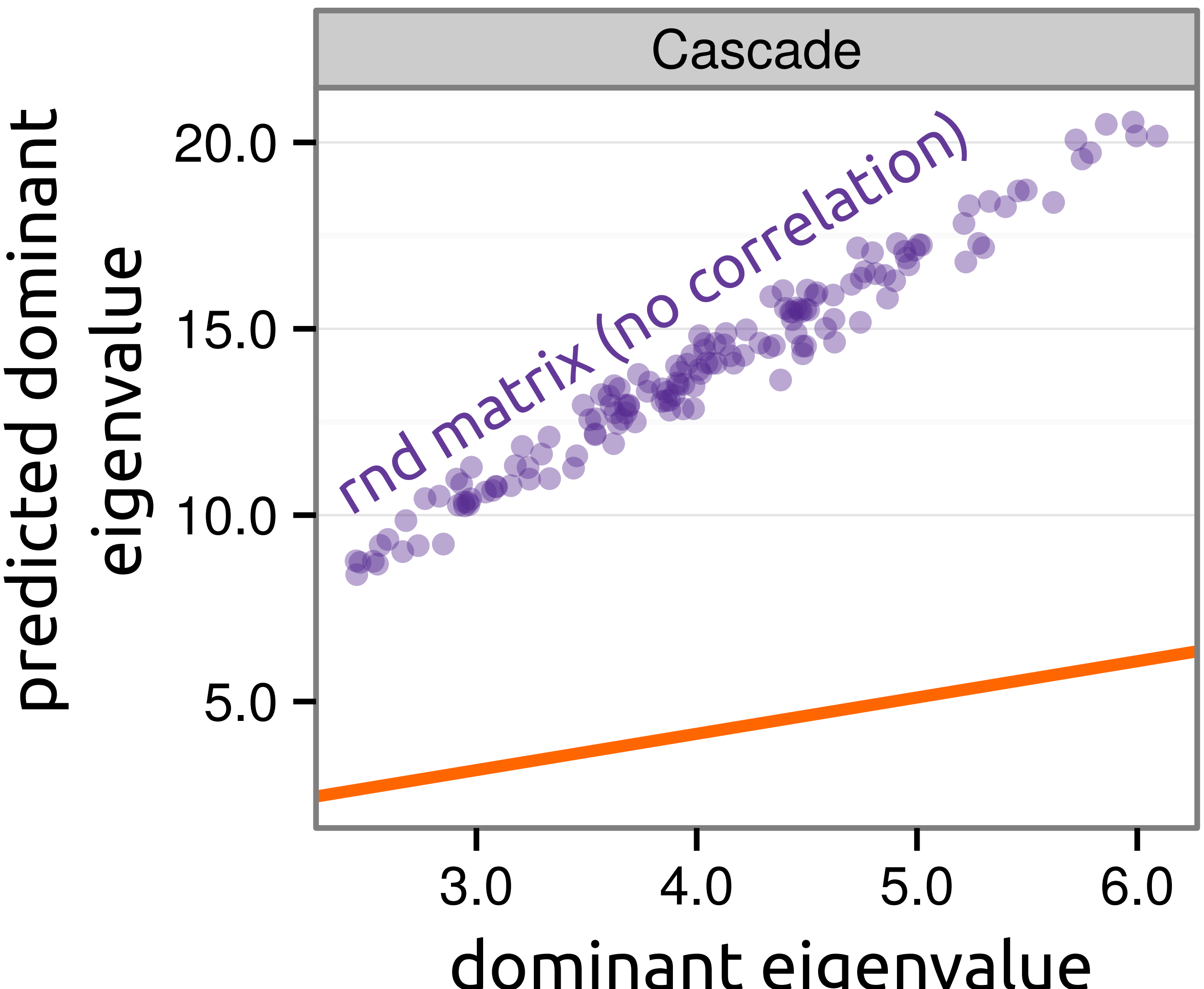


direction determined by the mean

Eigenvalues of B are uniform in an ellipse



It is possible to derive a new stability criterion for structured food-webs



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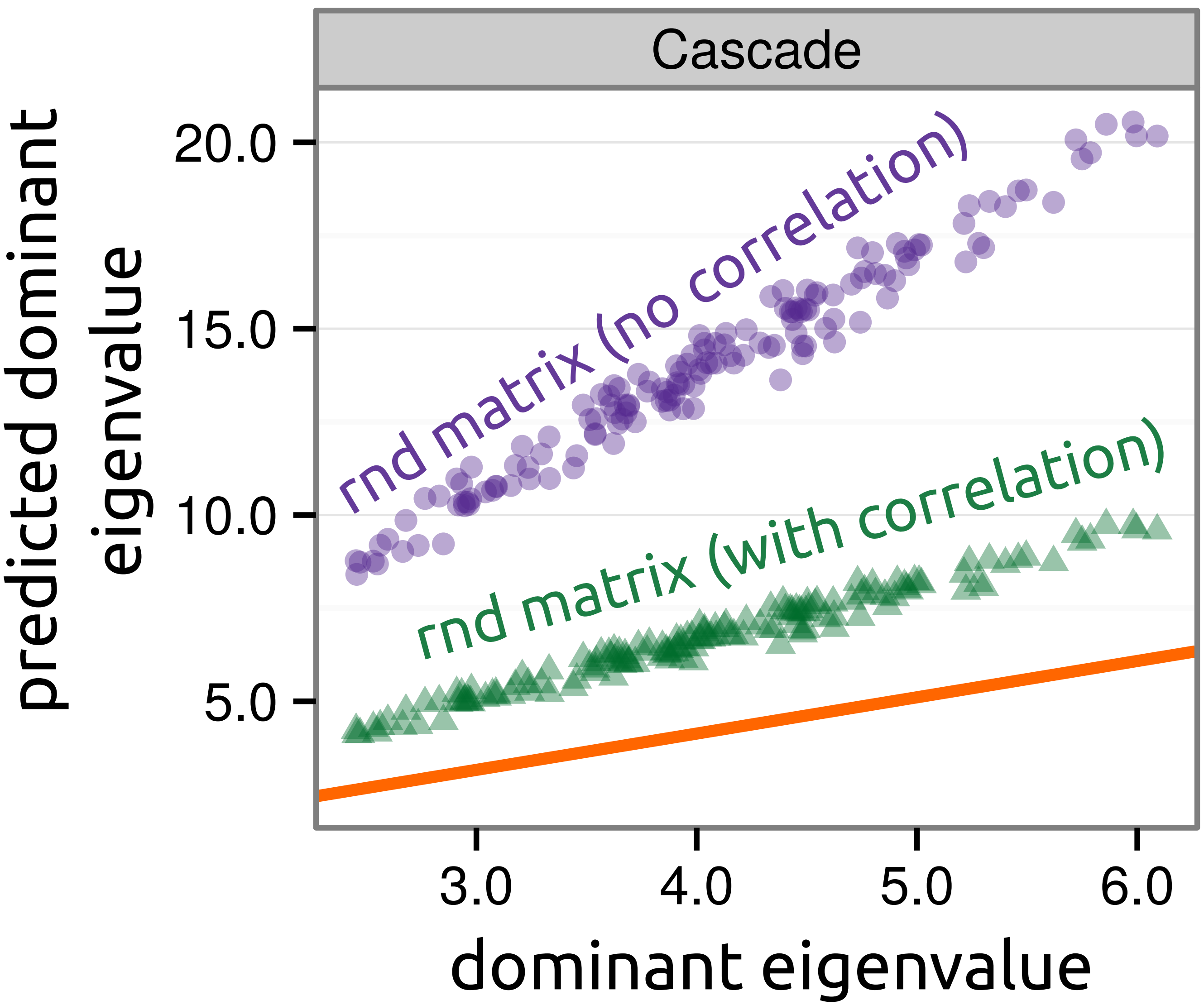

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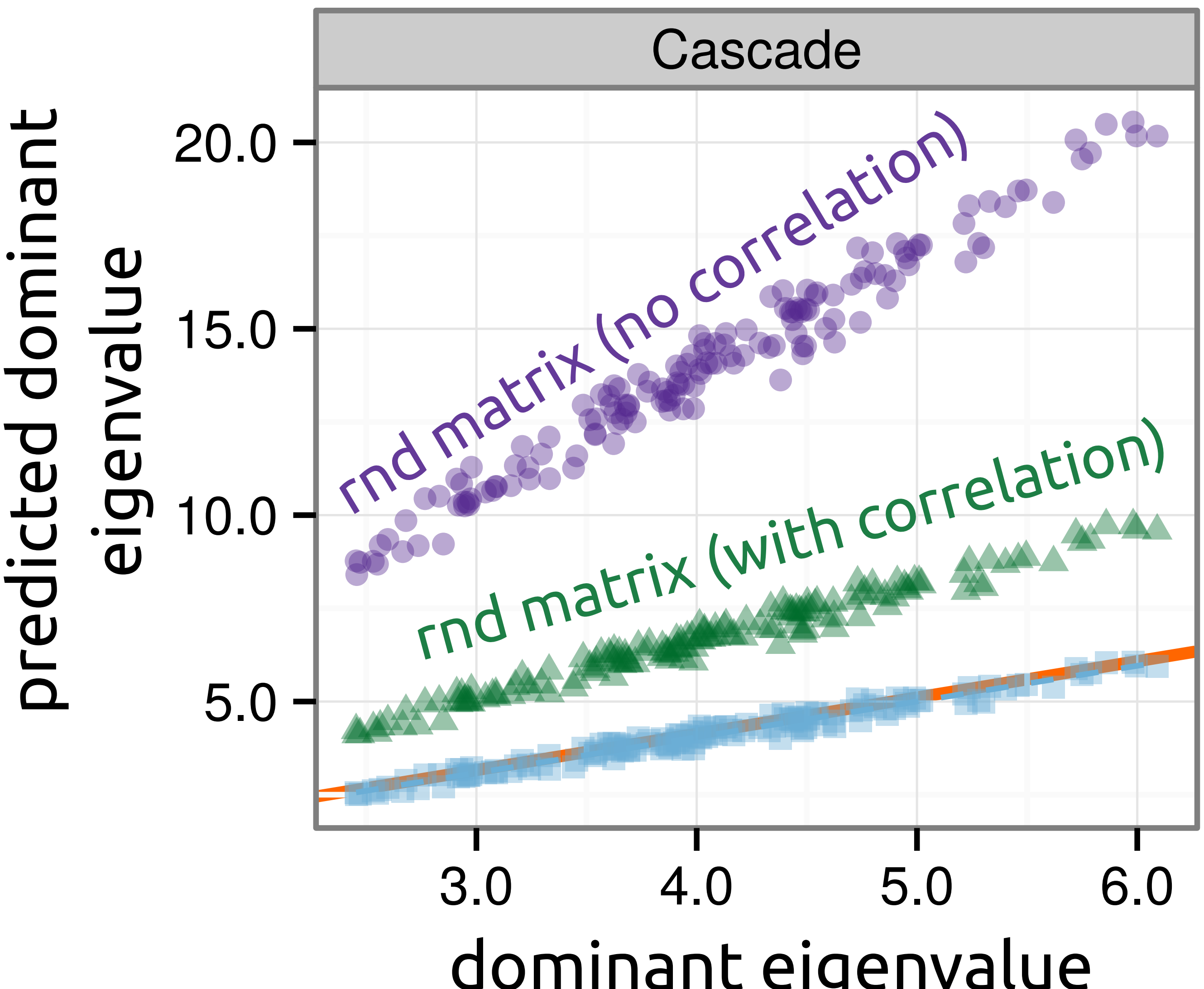

The stability criterion works well for empirical foodwebs

15 foodwebs
[empirical network structure]

coefficient determined using allometric scaling

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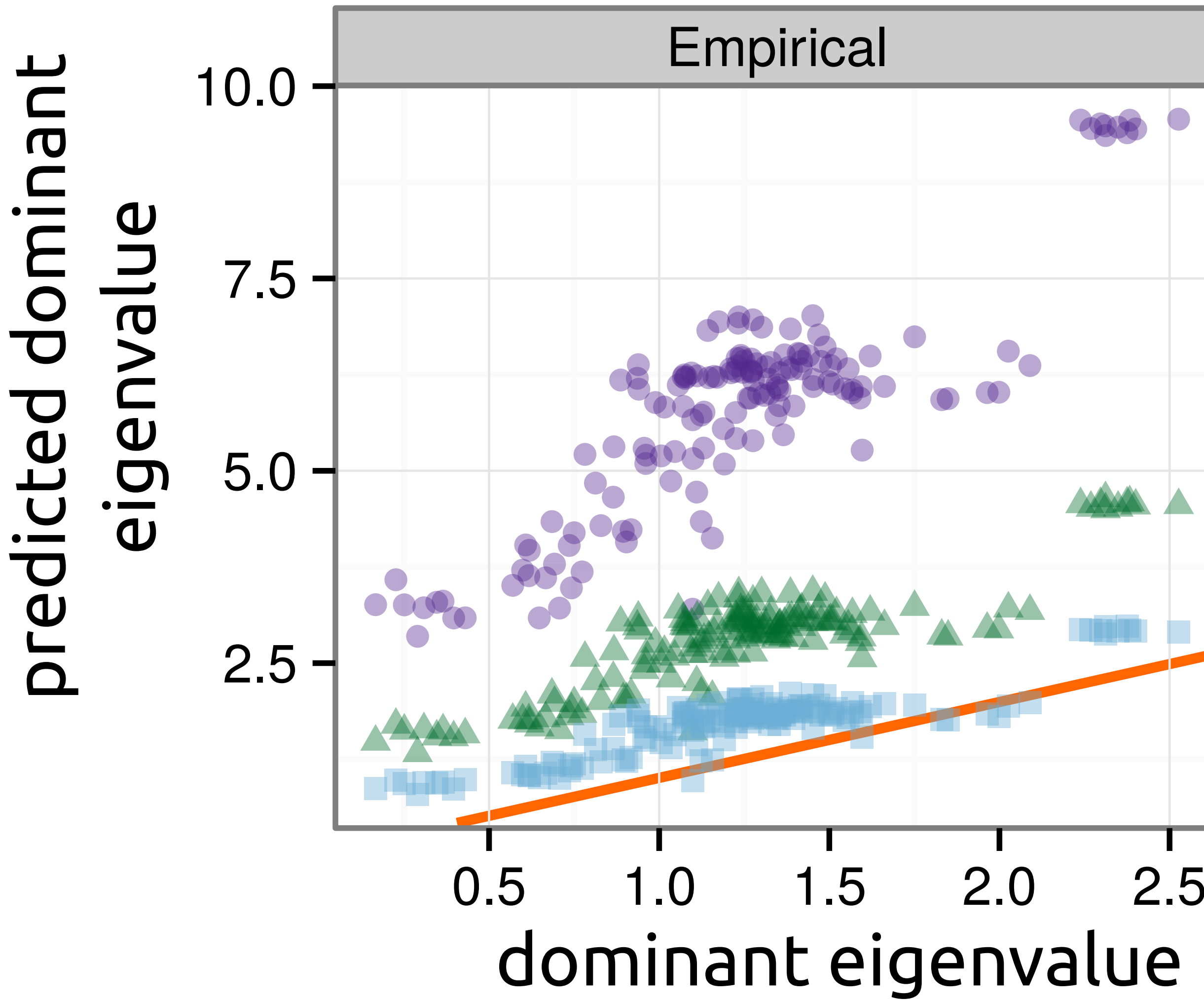

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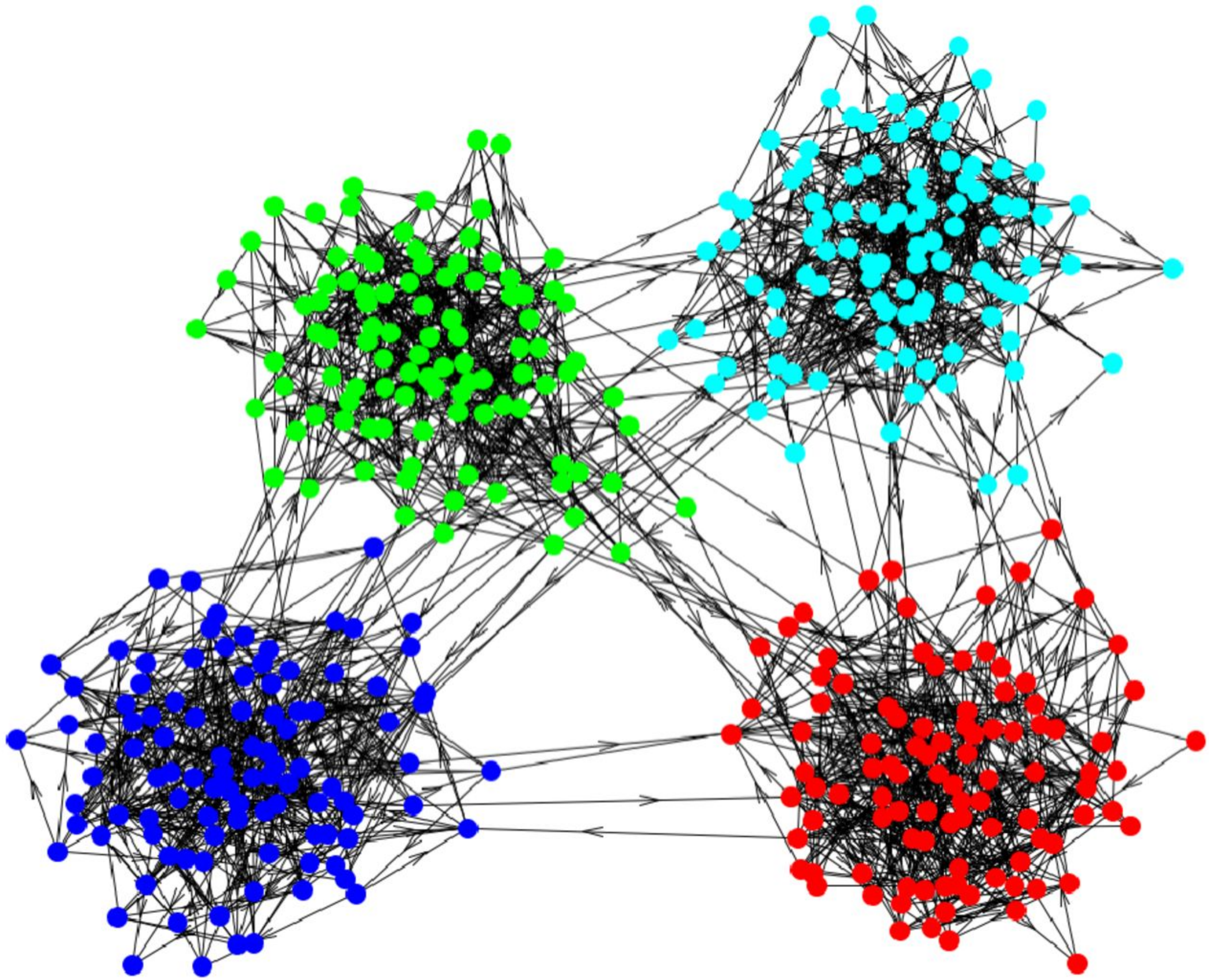
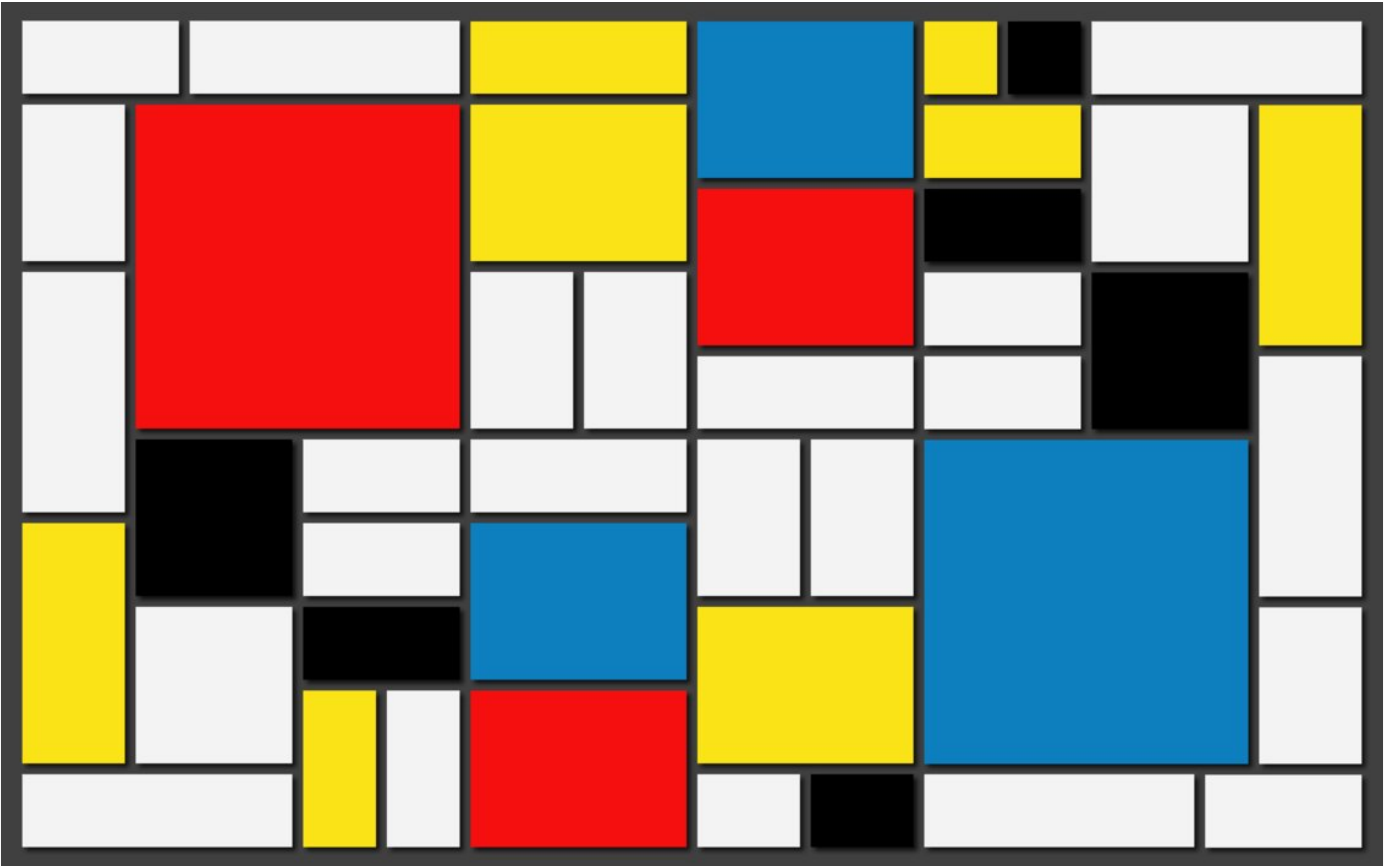
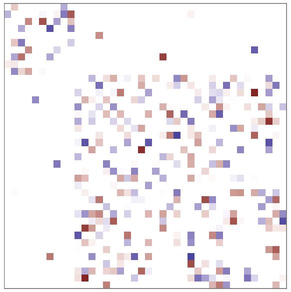
[Grilli, Rogers and Allesina, Nature Communications, 2016]

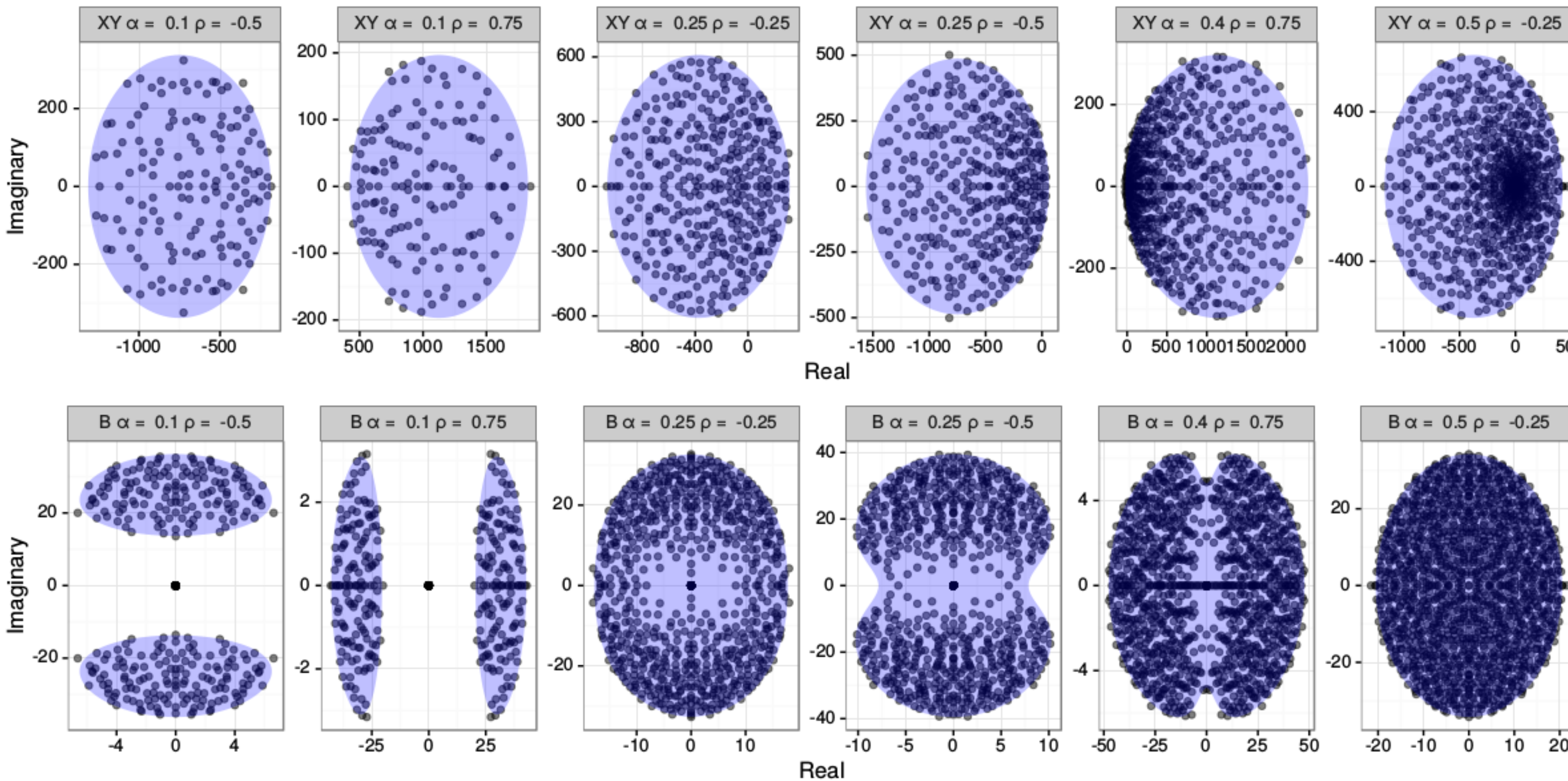
Communities





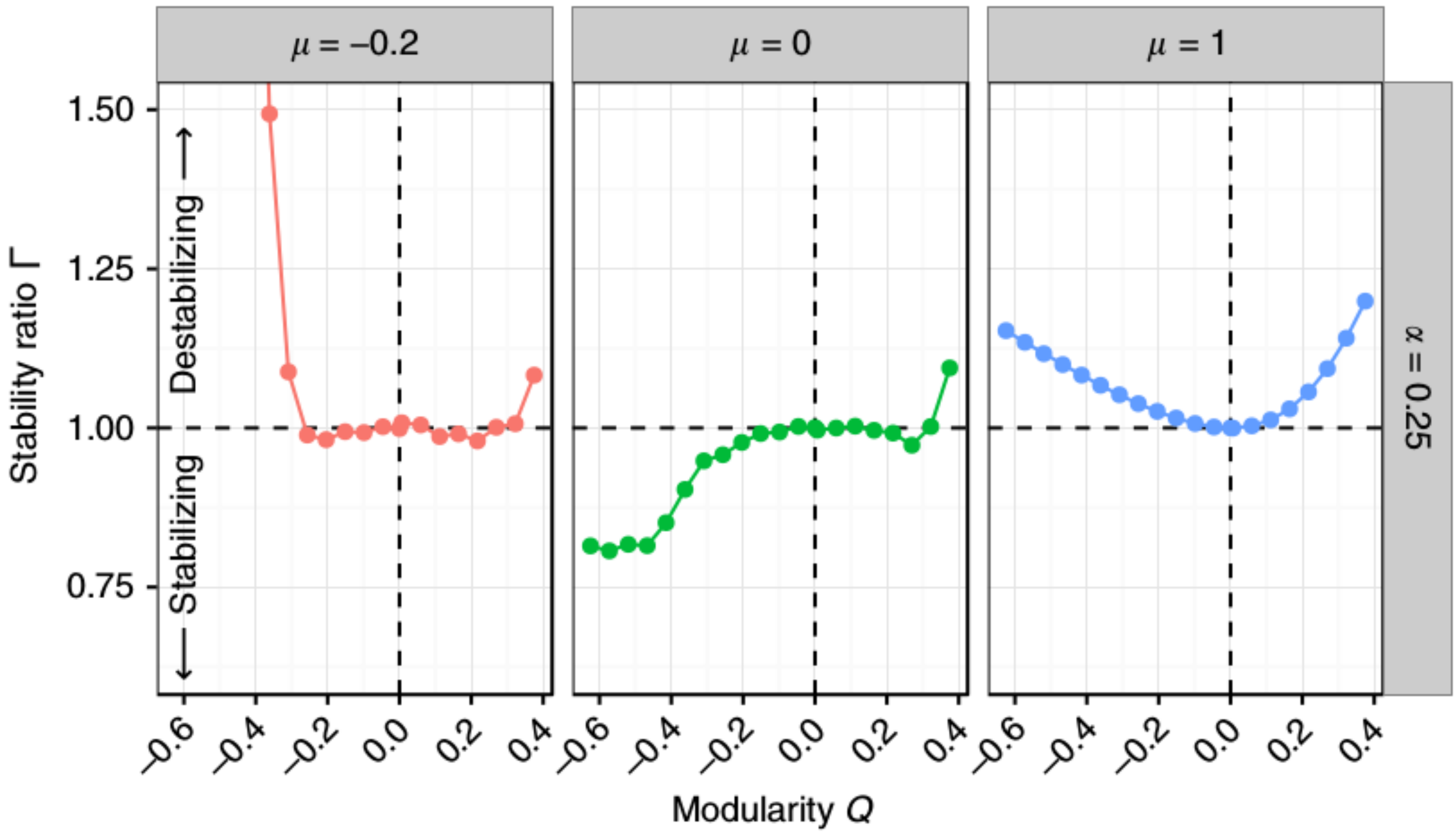
Full characterization of the effect of modularity



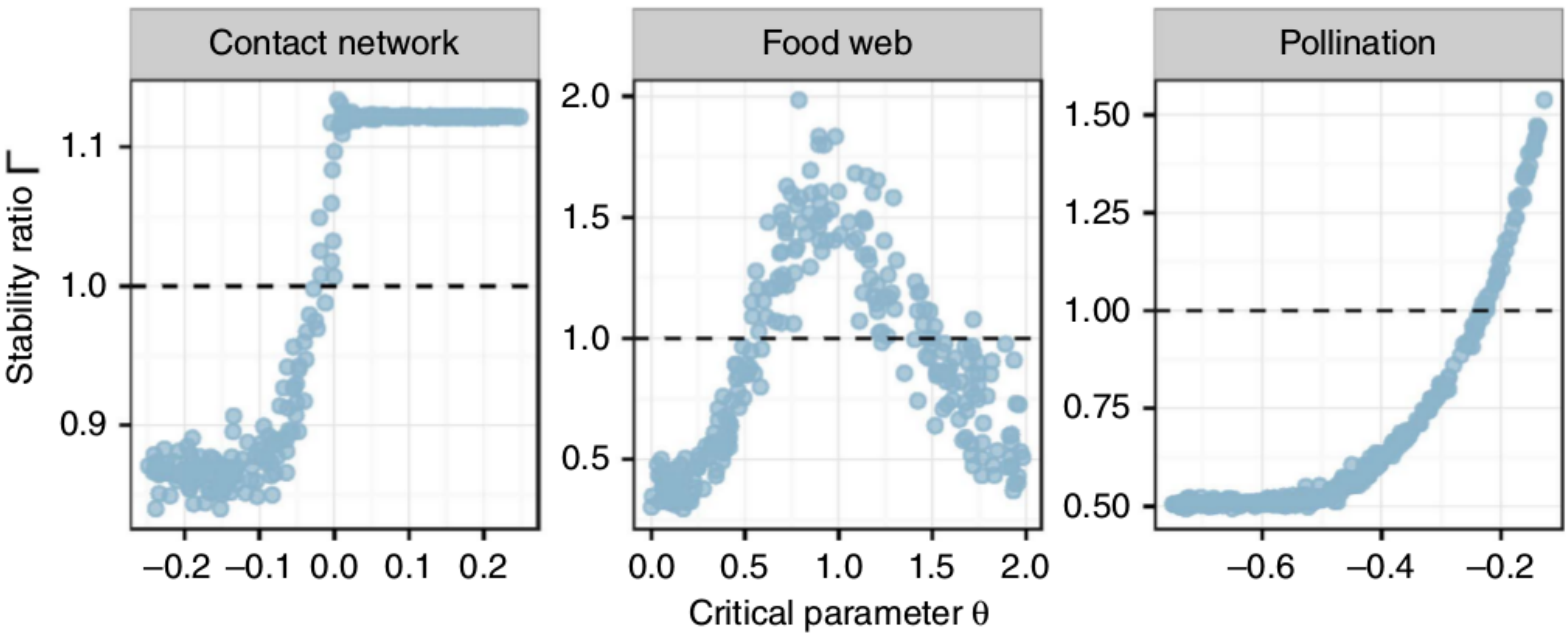


[Grilli, Rogers and Allesina, Nature Communications, 2016]

Usually destabilizing
(but effect depends on interactions)



"effect depends on interactions" is more general



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[Gibbs, Grilli, Rogers and Allesina, arXiv:1708.08837]

Stability in an explicit model

$$\frac{dx_i(t)}{dt} = \phi_i(x_i(t))H_i\left(\sum_j A_{ij}x_j(t)\right)$$

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Stability in an explicit model

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random matrix

$$\left.\frac{\partial f_i}{\partial x_j}\right|_{x^*} = M_{ij} = X_i A_{ij}$$

random vector

Stability in an explicit model

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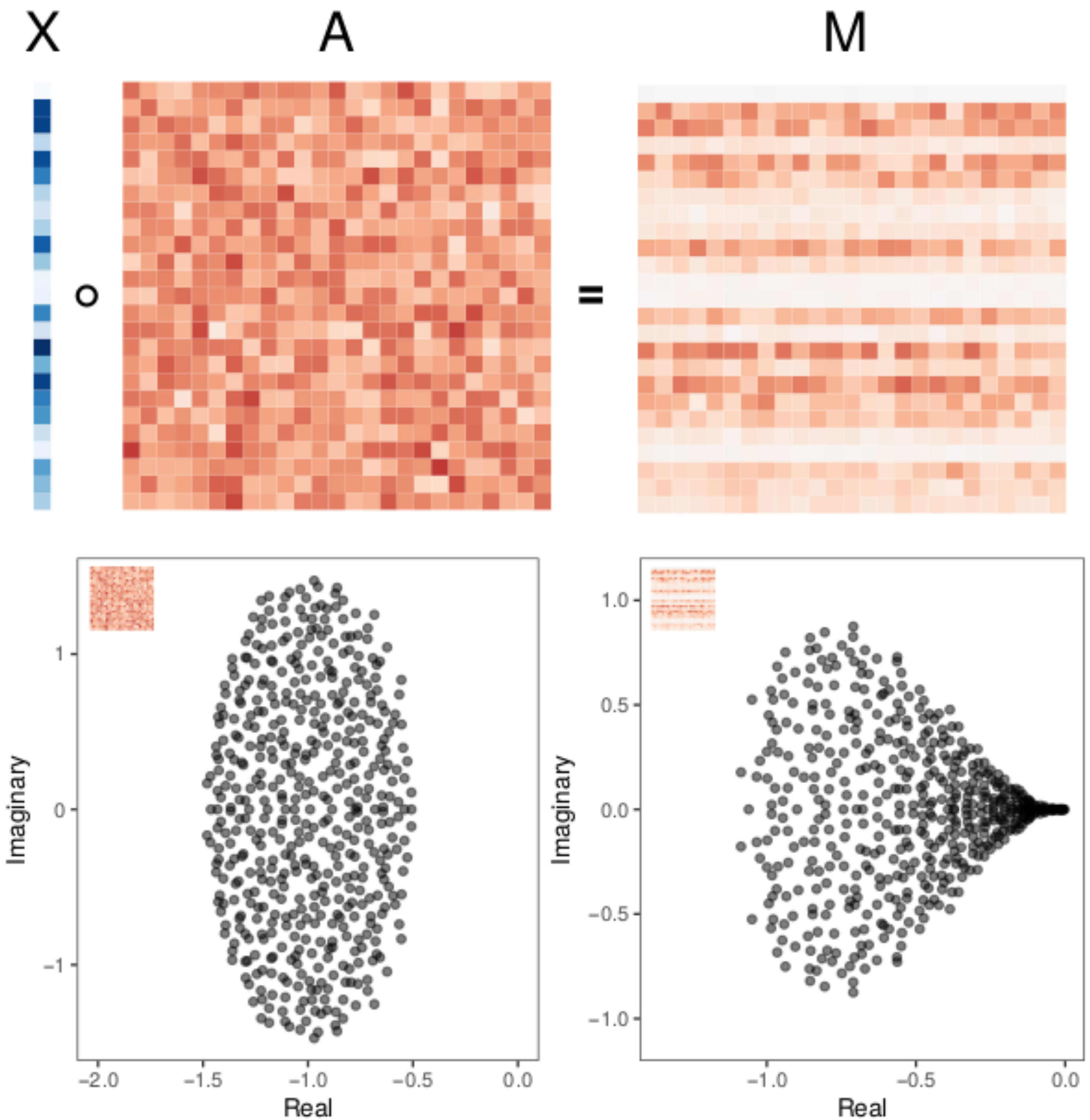

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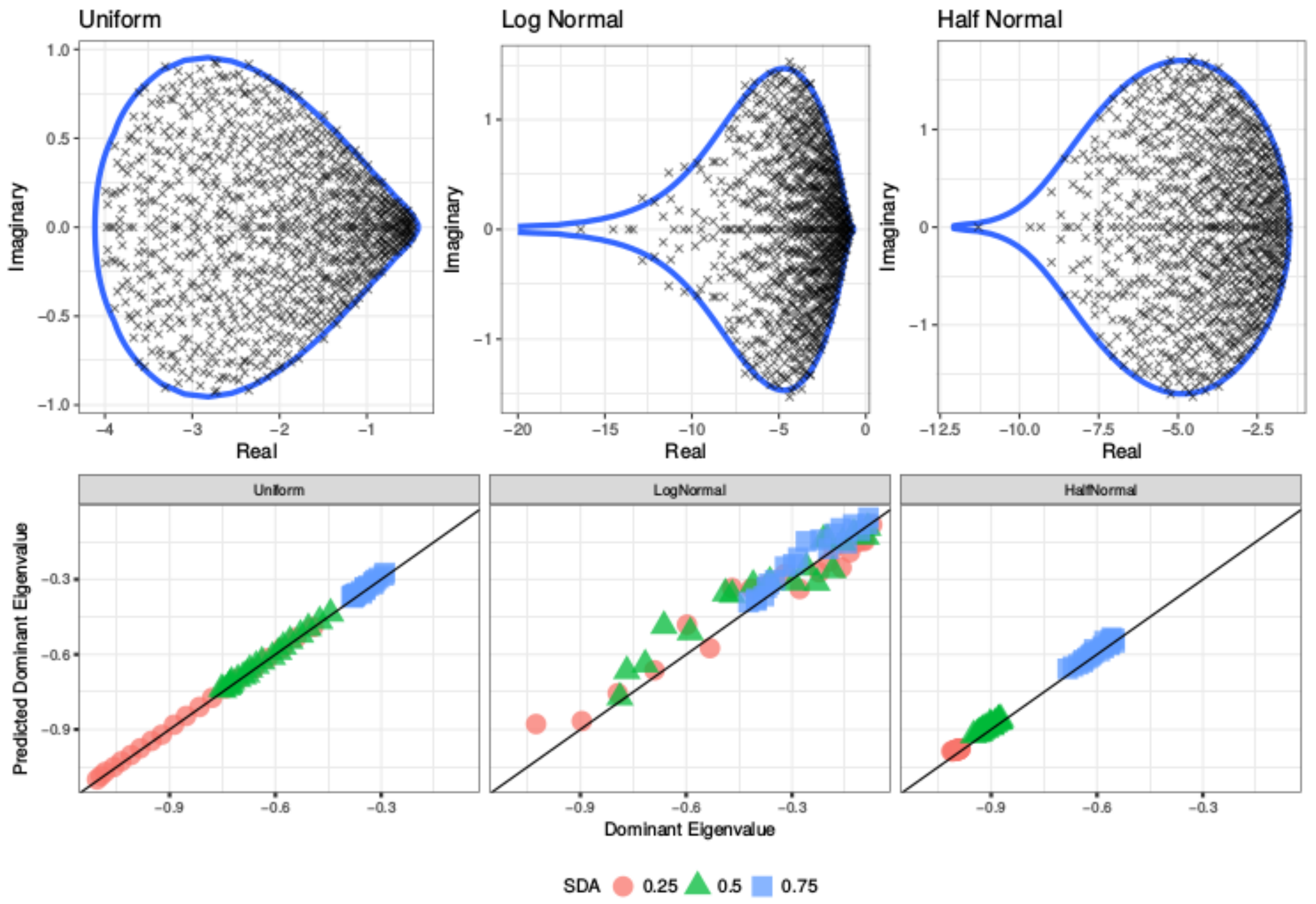
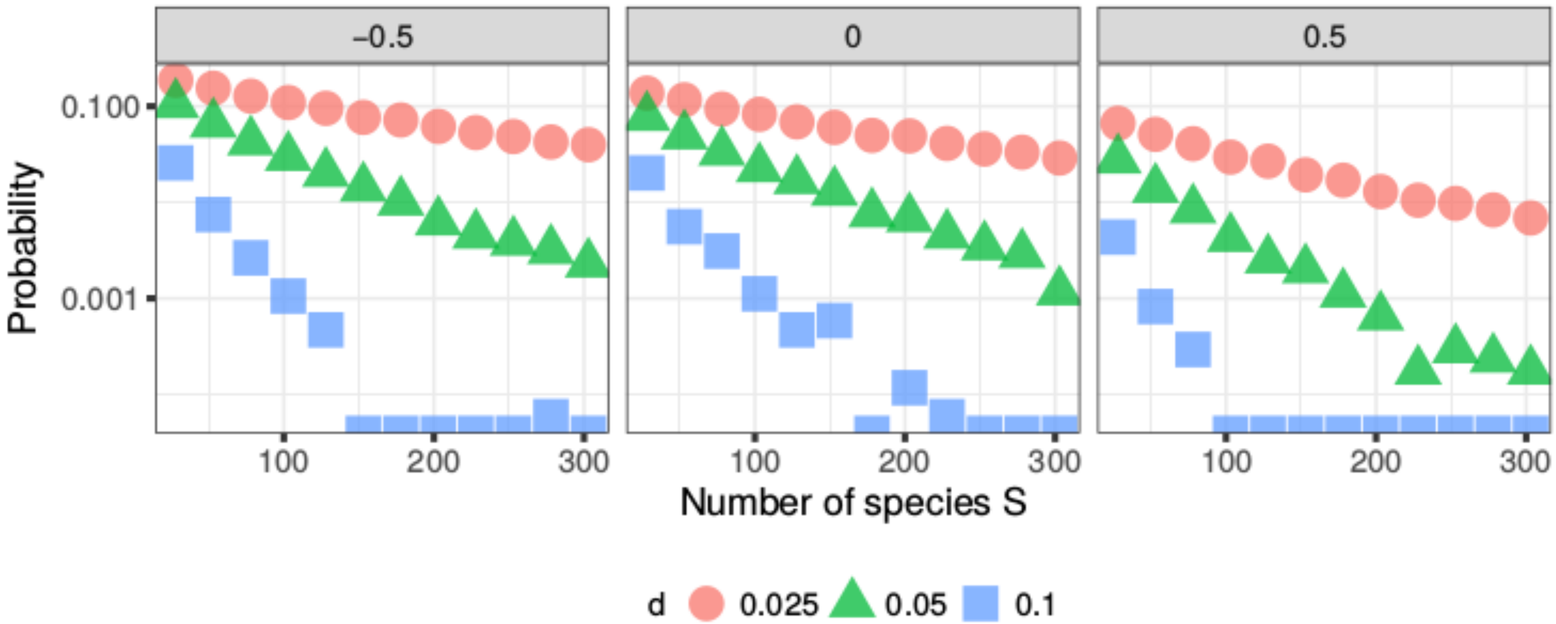
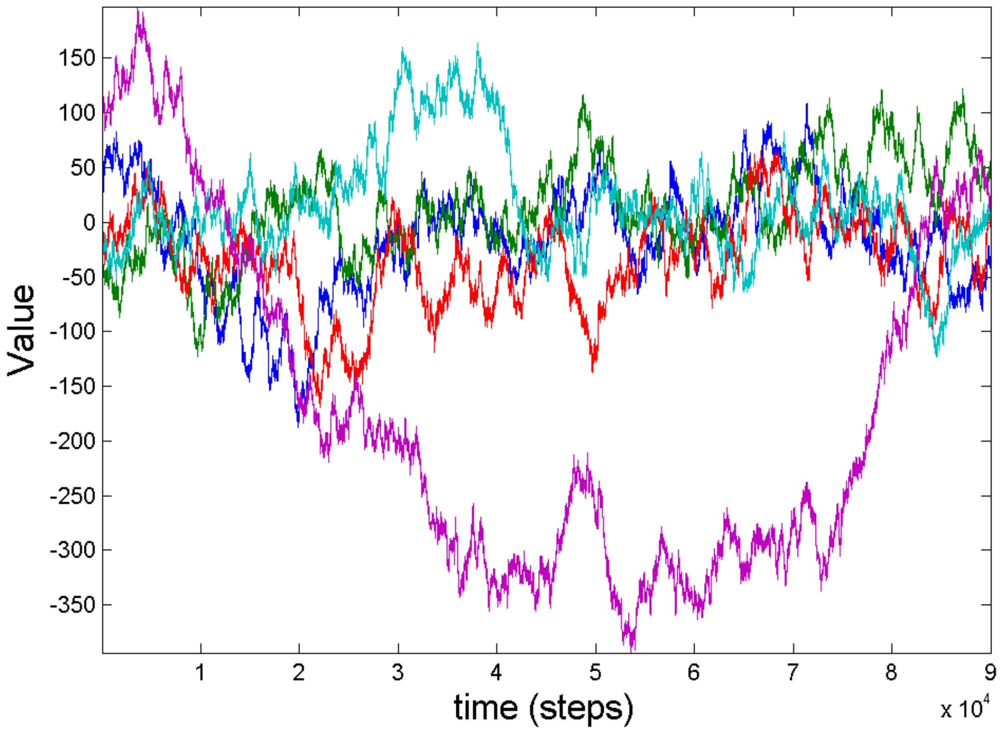


For large random matrices stability is determined uniquely by interactions



Measuring moments of interactions



Traditionally: infer N^2 interaction from N time series

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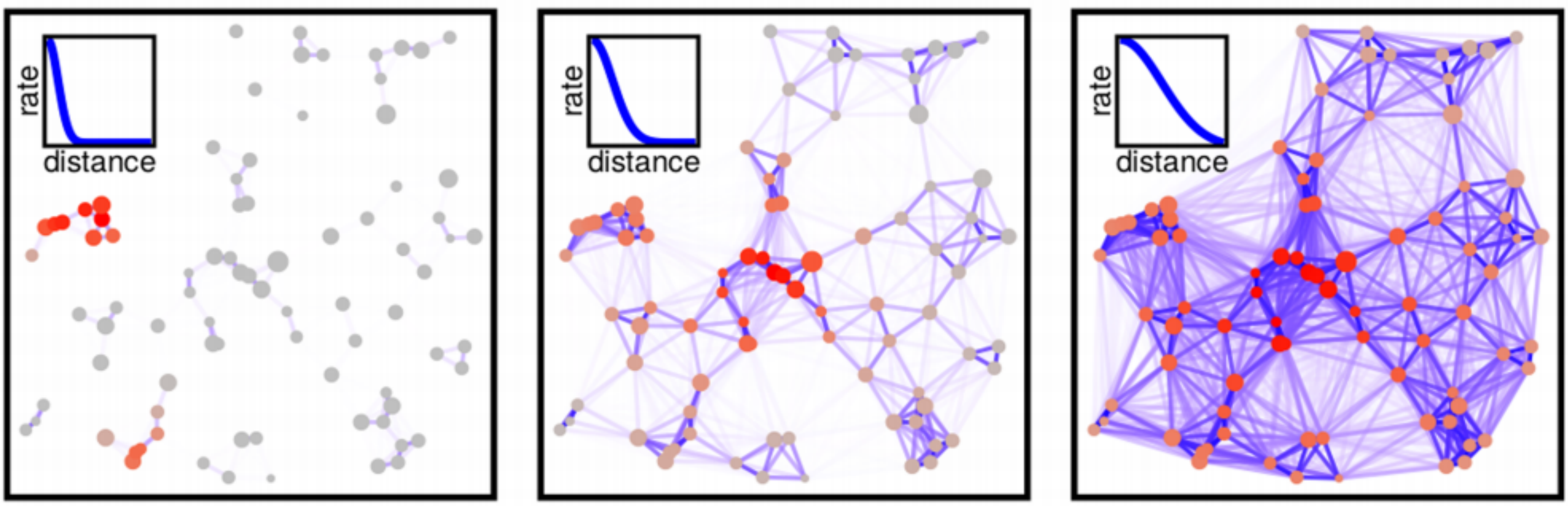
[Grilli, Barabais and Allesina, Plos Comp Bio 2015]

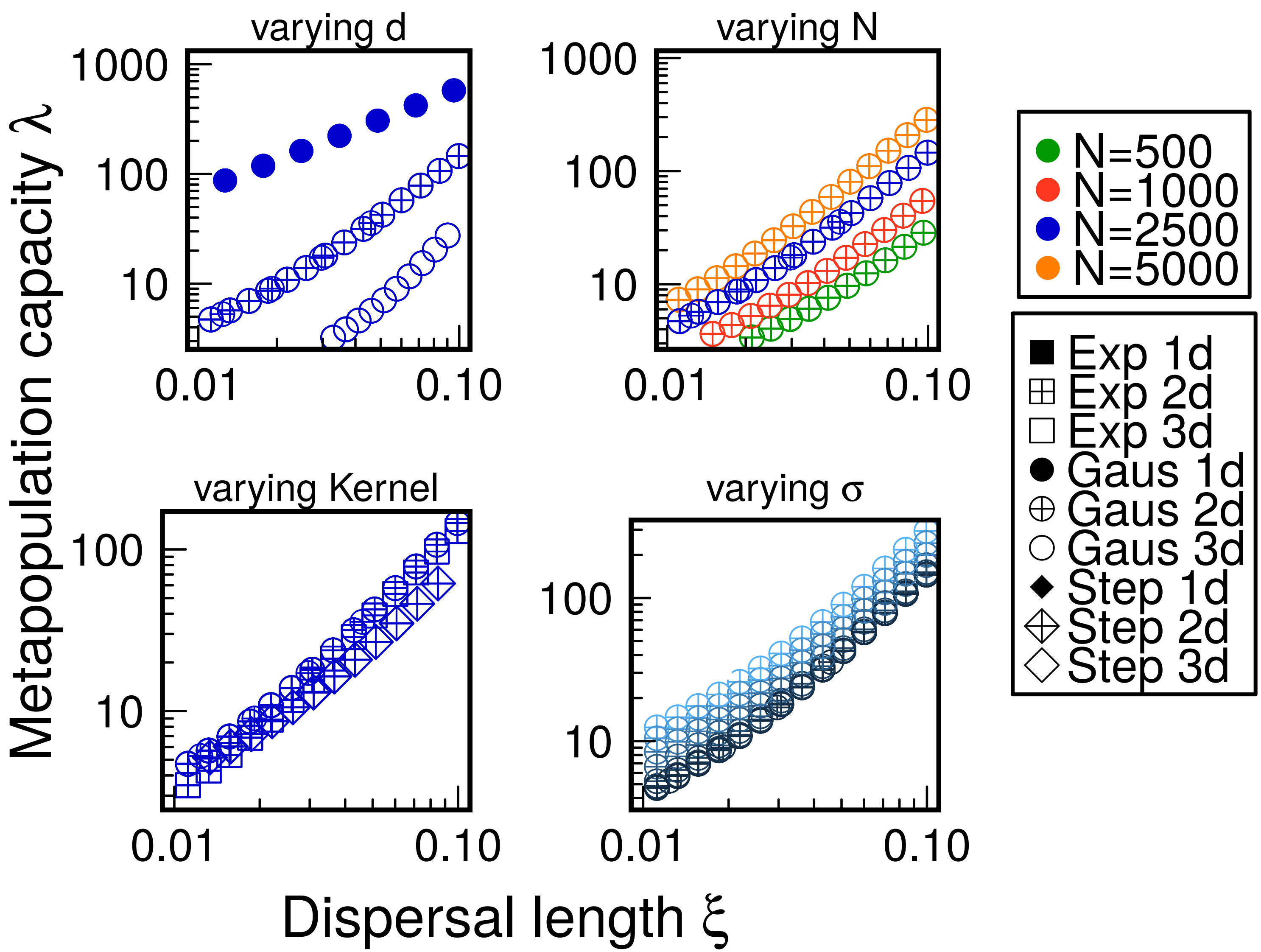
Metapopulation



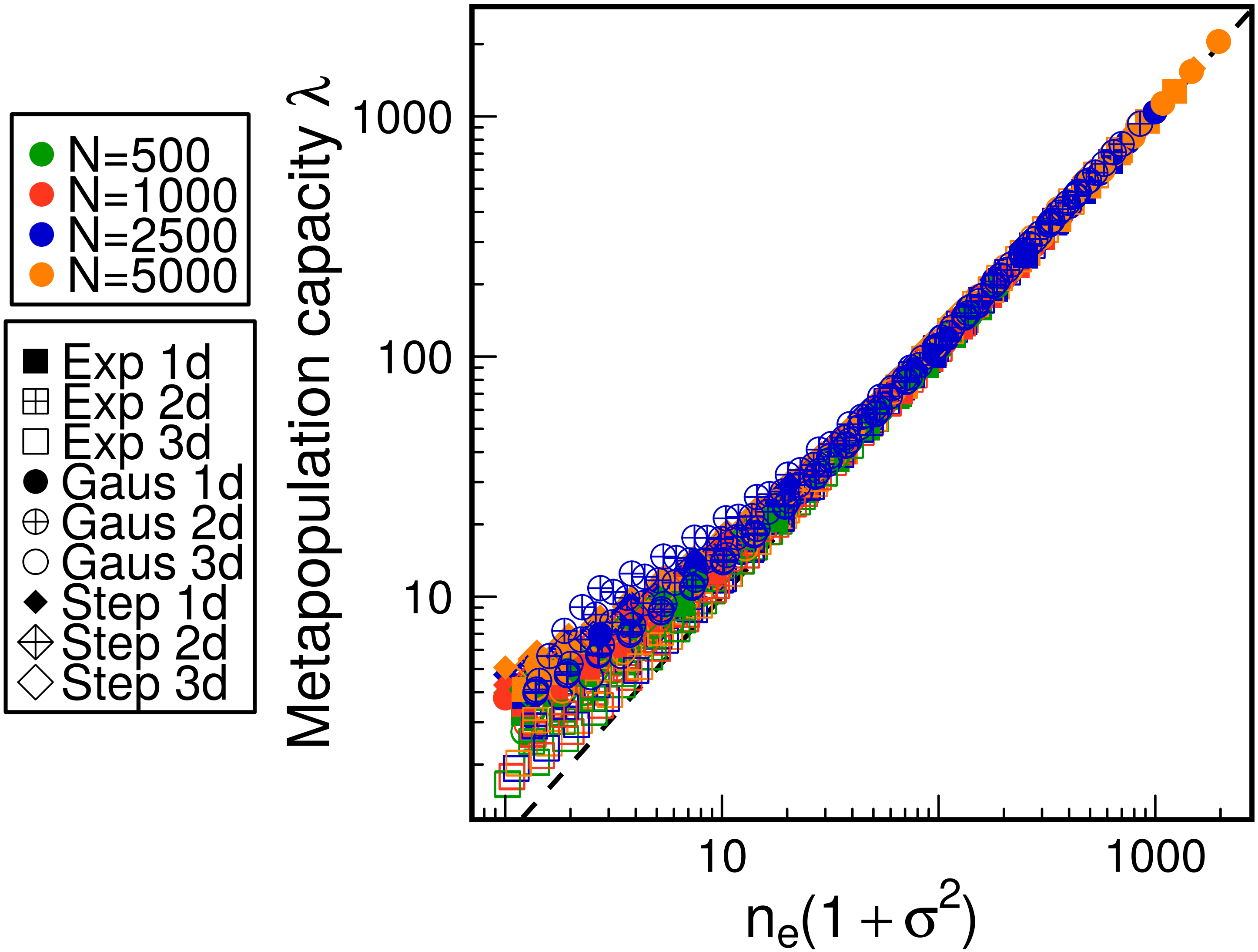
local death + dispersion

Persistence if eigenvalue of dispersion matrix is larger than death rate

Eigenvalue depends on everything



The eigenvalue depends on one effective parameter



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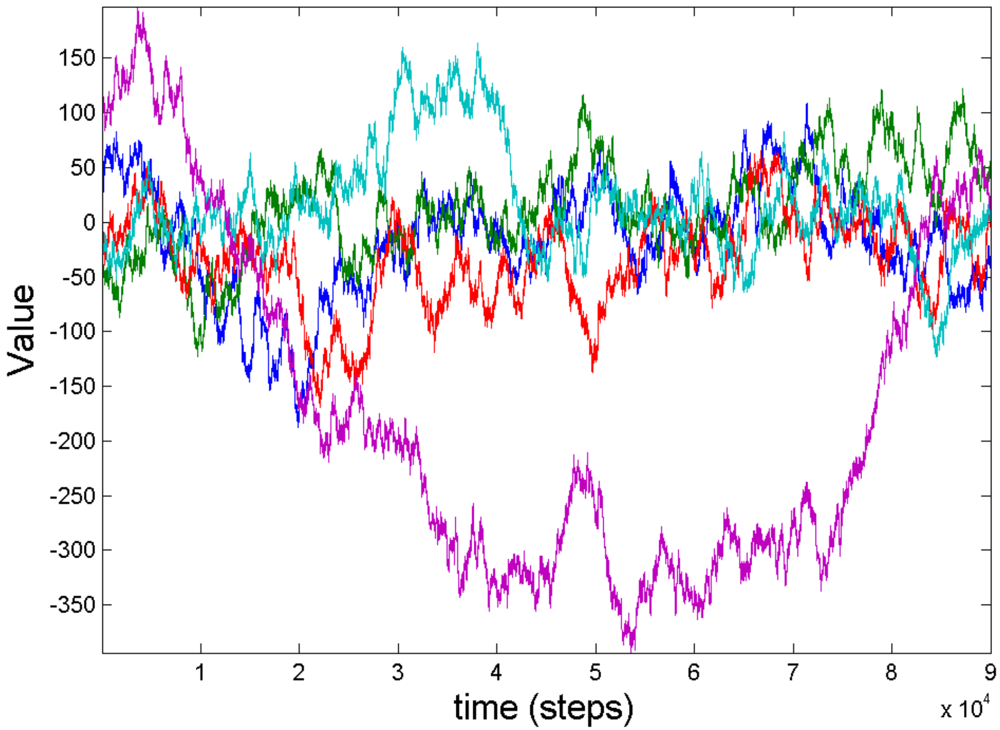

Traditionally: infer N^2 interaction from N time series

Can we directly infer the moments
(or more generally the statistical properties)
of the interactions?

Take home messages

Universality: lot of details do not matter

Few features of networks are important for stability

Network structure alone is not sufficient

Acknowledgments

S. Allesina, Y. Aljadeff, G. Barabás, T. Gibbs, T. Rogers, S. Tang